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Existence results for a coupled systems of Chandrasekhar quadratic integral equations

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Abstract

In this article, we study a coupled systems of generalized Chandrasekhar quadratic integral equations, which is widely applicable in various disciplines of science and technology. By using contraction mapping principle and successive approximation, we develop sufficient conditions for existence and uniqueness of solution. Also, an example is provided to illustrate our main results.

Keywords: Chandrasekhar quadratic integral equations, coupled system, contraction mapping principle, successive approximation method.
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1. Introduction

In this article, we develop sufficient conditions for existence and uniqueness of solution to the following coupled system of quadratic integral equations of Chandrasekhar's type given by

$$\begin{aligned} w(\tau) &= h_1(\tau) + g_1(\tau, w(\tau), x(\tau)) \int_0^1 \frac{\tau}{\tau + s} \Phi_1(s, w(s), x(s)) ds, \qquad \tau \in I = [0, 1], \\ x(\tau) &= h_2(\tau) + g_2(\tau, w(\tau), x(\tau)) \int_0^1 \frac{\tau}{\tau + s} \Phi_2(s, w(s), x(s)) ds, \qquad \tau \in I = [0, 1], \end{aligned} \tag{1.1}$$

where $h_k : [0, 1] \to [0, \infty)$ and $g_k, \Phi_k : [0, 1] \times D \times D \to [0, \infty)$, for $k = 1, 2$ are continuous functions. The aforementioned coupled system of integral equations is the generalization of the following generalized Chandrasekhar's quadratic integral equations provided by

$$w(\tau) = 1 + w(\tau) \int_0^1 \frac{\tau \lambda \phi(s)}{\tau + s} (log(1 + |w(s)|)) ds, \qquad \tau \in I = [0, 1], \tag{1.2}$$

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where ϕ is continuous function from $[0,1] \to [0,\infty)$ in [19]. The aforesaid integral equation was studied in many articles, see [10, 9]. Integral equations is attractive area of research in past as well as in recent times. This is due to the fact that the integral equations has many applications in applied sciences and technology, (for detail see [3, 12, 4, 5, 6, 7]). This system used in many problems of applied science, (see[1, 8, 11, 15, 16, 13, 14, 17]). Su [20], proved a two-point boundary value problem for a coupled system of fractional differential equations. Gafiychuk *et al.* [17], discussed the solution of coupled nonlinear fractional-diffusion equations. Moreover in recent times most of the biological and physical models are in the form of integral equations or their systems. Therefore the concerned area attracted much attention from researchers. The Chandrasekhar quadratic integral equations are used in studying of connection with scatting through a homogenous semi-infinite atmosphere [10]. In astrophysical physical applications of Chandrasekhar quadratic integral equation the only restriction is that $\int_0^1 \phi(s)ds \leq \frac{1}{2}$ is a necessary conditions in [9]. Applications of quadratic integral equations are in the kinetic theory of gasses, in theory of neutron transport, in theory of radiative transfer and the traffic theory. The Chandrasekhar quadratic integral equations have many applications [2]. Several authors proved the existence of solutions for nonlinear quadratic integral equations (see [3, 12, 4, 5, 6, 7]). From all of the above literature, the main results are obtained with the help of the methods which are related to the measure of non compactness. In [18], used fixed point theorem to prove the existence of solution of some quadratic integral equations.Due to these importance and uses, we study [2], system of integral equation. The concern study is carried out with the help of fixed point theorem of Banach contraction type. Moreover for approximating the solution, we apply monotone iterative techniques of Picard's type successive approximation procedure to develop sufficient condition for approximating the solutions. Further, we also give an example to verify our main results.

2. Preliminaries

In this section, we give some assumptions which are needed throughout this paper.

(A_1) $h_k : [0,1] \to [0,\infty), k = 1,2$ are continuous on $[0,1]$.

(A_2) $g_k, \Phi_k : [0,1] \times D \times D \to [0,\infty), k = 1,2$ are continuous, where $D \subseteq [0,\infty)$.

(A_3) There exists positive constants M_k and $N_k, k = 1,2$ such that $|g_k(\tau,w,x)| \leq M_k$ and $|\Phi_k(\tau,w,x)| \leq N_k$ for $(\tau,w,x) \in I \times D \times D$.

(A_4) $\Phi_k,\ g_k$, for $k = 1,2$ satisfy the Lipschitz condition with Lipschitz constants $L_k,\ K_k$, such that

$$|g_k(\tau,w_k,x_k) - g_k(\tau,\bar{w}_k,\bar{x}_k)| \leq L_k\left[|w_k - \bar{w}_k| + |x_k - \bar{x}_k|\right]$$

and

$$|\Phi_k(\tau,w_k,x_k) - \Phi_k(\tau,\bar{w}_k,\bar{x}_k)| \leq K_k\left[|w_k - \bar{w}_k| + |x_k - \bar{x}_k|\right].$$

Let $X = C[0,1]$ be the class of all real continuous function defined and continuous on $[0,1]$ with the norm $||w|| = \max|w(\tau)| : \tau \in [0,1]$. Then the norm in product space be defined by $||(w,x)|| = ||w|| + ||x||$.

3. Main Result

Defined the operator by

$$T(w,x)(\tau) = (T_1(w,x), T_2(w,x))(\tau),$$

where

$$T_1(w,x)(\tau) = h_1(\tau) + g_1(\tau,w(\tau),x(\tau))\int_0^1 \frac{\tau}{\tau+s}\Phi_1(s,w(s),x(s))ds, \qquad \tau \in I = [0,1],$$

$$T_2(w,x)(\tau) = h_2(\tau) + g_2(\tau,w(\tau),x(\tau))\int_0^1 \frac{\tau}{\tau+s}\Phi_2(s,w(\tau),x(s))ds, \qquad t \in I = [0,1].$$

Theorem 3.1. *Let assumptions* $(A_1)-(A_3)$ *hold. Further, if*

$$M_1K_1 + L_1N_1 + M_2K_2 + L_2N_2 < 1, \ \textit{for } k = 1, 2.$$

Then the coupled system (7) *has a unique solution.*

Proof. Define $S = \{||(w, x)|| \leq r : (w, x)(\tau) \in X \times X\}$. Then

$$\begin{aligned}|T_1(w_2, x_2) - T_1(w_1, x_1)| &\leq \left| g_1(\tau, w_2, x_2) \int_0^\tau \frac{\tau}{\tau + s} \Phi_1(s, w_2, x_2) ds - g_1(\tau, w_1, x_1) \int_0^\tau \frac{\tau}{\tau + s} \Phi_1(s, w_1, x_1) ds \right| \\ &\leq |g_1(\tau, w_1, x_1)| \int_0^\tau \left| \frac{\tau}{\tau + s} \right| |\Phi_1(s, w_1(s), x_1(s)) - \Phi_1(s, w_2(s), x_2(s))| ds \\ &+ |g_1(\tau, w_2, x_2) - g_1(\tau, w_1, x_1)| \int_0^\tau \left| \frac{\tau}{\tau + s} \right| |\Phi_1(s, w_2(s), x_2(s))| ds \\ &\leq M_1K_1 [||w_1 - w_2|| + ||x_1 - x_2||] + L_1N_1 [||w_1 - w_2|| + ||x_1 - x_2||] .\end{aligned} \tag{3.1}$$

So,

$$||T_1(w_1, x_1) - T_1(w_2, x_2)|| \leq M_1K_1 [||w_1 - w_2|| + ||x_1 - x_2||] + L_1K_1 [||w_1 - w_2|| + ||x_1 - x_2||] ,$$

which implies that

$$\|T_1(w_1, x_1) - T_1(w_2, x_2)\| \leq (M_1K_1 + L_1K_1) [\|w_1 - x_1\| + \|w_2 - x_2\|] .$$

Similarly, one can also has

$$||T_2(w_1, x_1) - T_2(w_2, x_2)|| \leq (M_2K_2 + L_2K_2) [||w_1 - x_1|| + ||w_2 - x_2||] . \tag{3.2}$$

Now, from (3.1) and (3.2), we have

$$\|T(w_1, x_1) - T(w_2, x_2)\| \leq ((M_1K_1 + L_1K_1) + (M_2K_2 + L_2K_2)) [||w_1 - x_1|| + ||w_2 - x_2||] . \tag{3.3}$$

Which implies that T is contraction. Hence the coupled system (7) has a unique solution by contraction principle. This end the proof. □

4. Method of successive approximation

Theorem 4.1. *Let the assumption* $(A_1)-(A_3)$ *be satisfied and there exists monotone sequences* $w_n(\tau), x_n(\tau)$ *such that* $w_n(\tau) \to w(\tau)$ *and* $x_n(\tau) \to x(\tau), \tau \in [0, 1]$ *as* $n \to \infty$ *and this convergence is uniformly and monotonically on* $[0, 1]$.

Proof. Since T_1 and $T_2 : X \times X \to X \times X$. Then let us consider two sequences corresponding to the coupled system of quadratic integral equation (4) as

$$\begin{aligned} w_n(\tau) &= h_1(\tau) + g_1(\tau, w_{n-1}(\tau), x_{n-1}(\tau)) \\ &\quad \times \int_0^1 \frac{\tau}{\tau + s} \Phi_1(s, w_{n-1}(s), x_{n-1}(s)) ds, \ \tau \in I = [0, 1], \\ x_n(\tau) &= h_2(\tau) + g_2(\tau, w_{n-1}(\tau), x_{n-1}(\tau)) \\ &\quad \times \int_0^1 \frac{\tau}{\tau + s} \Phi_2(s, w_{n-1}(s), x_{n-1}(s)) ds, \ \tau \in I = [0, 1]. \end{aligned} \tag{4.1}$$

Also, when $n = 0$, $w_0 = h_1(\tau), x_0 = h_2(\tau)$. As $w_n(\tau)$ and $x_n(\tau)$ are continuous functions, then in view of Picard successive method, $w_n(\tau)$ and $x_n(\tau)$ can be written as a sum of successive differences as given by

$$w_n = w_0 + \sum_{i=1}^{n}(w_i - w_{i-1}),\ x_n = x_0 + \sum_{i=1}^{n}(x_i - x_{i-1}).$$

Thus convergence of w_n and x_n implies convergence of two series

$$\sum_{i=1}^{n}(w_i - w_{i-1}) \text{ and } \sum_{i=1}^{n}(x_i - x_{i-1})$$

and the correspondence solution will be

$$w(\tau) = \lim_{n\to\infty} w_n(\tau) \text{ and } x(\tau) = \lim_{n\to\infty} x_n(\tau). \tag{4.2}$$

For uniform convergence consider the following infinite series using $n = 2$ in (4.2), we get

$$\sum_{i=1}^{\infty}|w_n(\tau) - w_{n-1}(\tau)|,\ \sum_{i=1}^{\infty}|x_n(\tau) - x_{n-1}(\tau)|.$$

From (4.1), we have for $n = 1$

$$\begin{aligned}
w_1(\tau) - w_0(\tau) &= g_1(\tau, w_0(\tau), x_0(\tau)) \int_0^1 \frac{\tau}{\tau+s}\Phi_1(s, w_0(s), x_0(s))ds, \qquad \tau \in I = [0,1],\\
x_1(\tau) - x_0(\tau) &= g_2(\tau, w_0(\tau), x_0(\tau)) \int_0^1 \frac{\tau}{\tau+s}\Phi_2(s, w_0(s), x_0(s))ds, \qquad \tau \in I = [0,1].
\end{aligned}$$

From which we have

$$||w_1 - w_0|| \le M_1N_2,\ \text{and } ||x_1 - x_0|| \le M_1N_2.$$

Now by induction, we obtain approximation for $n \ge 2$, as

$$\begin{aligned}
|w_n(\tau) - w_{n-1}(\tau)| \le \Big| & g_1(\tau, w_{n-1}(\tau), x_{n-1}(\tau)) \int_0^1 \frac{\tau}{\tau+s}\Phi_1(s, w_{n-1}(s), x_{n-1}(s))ds\\
& - g_1(\tau, w_{n-2}(\tau), x_{n-2}(\tau)) \int_0^1 \frac{\tau}{\tau+s}\Phi_1(s, w_{n-2}(s), x_{n-2}(s))ds\\
& + g_1(\tau, w_{n-2}(\tau), x_{n-2}(\tau)) \int_0^1 \frac{\tau}{\tau+s}\Phi_1(s, w_{n-2}(s), x_{n-2}(s))ds\\
& - g_1(\tau, w_{n-2}(\tau), x_{n-2}(\tau)) \int_0^1 \frac{\tau}{\tau+s}\Phi_1(s, w_{n-2}(s), x_{n-2}(s))ds\Big|\\
& \le |g_1(\tau, w_{n-2}(\tau), x_{n-2}(\tau))| \int_0^1 \left|\frac{\tau}{\tau+s}\right|\\
& \times \Big|[\Phi_1(s, w_{n-1}(s), x_{n-1}(s)) - \Phi_1(s, w_{n-2}(s), x_{n-2}(s))]\Big|ds\\
& + \Big|g_1(\tau, w_{n-1}(\tau), x_{n-1}(\tau)) - g_1(\tau, w_{n-2}(\tau), x_{n-2}(\tau))\Big|\\
& \times \int_0^1 \left|\frac{\tau}{\tau+s}\right||\Phi_1(s, w_{n-1}(s), x_{n-1}(s))|ds.
\end{aligned} \tag{4.3}$$

Which in view of A_2 and A_3, we get

$$\begin{aligned}
|w_n(\tau) - w_{n-1}(\tau)| \le M_1 & \int_0^1 \left|\frac{\tau}{\tau+s}\right| K_1\left[|w_{n-1}(s), w_{n-2}(s)| + ||x_{n-1}(\tau) - x_{n-2}(\tau)||\right] ds\\
& + L_1\left[|w_{n-1}(s), u_{n-2}(s)| + |x_{n-1}(\tau) - x_{n-2}(\tau)|\right] N_1 \int_0^1 \left|\frac{\tau}{\tau+s}\right|ds
\end{aligned} \tag{4.4}$$

and

$$\begin{aligned}|x_n(\tau)-x_{n-1}(\tau)| \le M_2\int_0^1\left|\frac{\tau}{\tau+s}\right|K_2\left[|w_{n-1}(s),w_{n-2}(s)|+||x_{n-1}(\tau)-x_{n-2}(\tau)||\right]ds\\ +L_2\left[|w_{n-1}(s),w_{n-2}(s)|+|x_{n-1}(\tau)-x_{n-2}(\tau)|\right]N_2\int_0^1\left|\frac{\tau}{\tau+s}\right|ds.\end{aligned}\tag{4.5}$$

Now for $n=2$ in (4.4) and (4.5), we have

$$\begin{aligned}|w_2(\tau)-w_1(\tau)| \le M_1\int_0^1\left|\frac{\tau}{\tau+s}\right|K_1\left[|w_1(s)-w_0(s)|+|x_1(s)-x_0(s)|\right]ds\\ +L_1\left[|w_1(s),w_0(s)|+|x_1(s)-x_0(s)|\right]N_1\int_0^1\left|\frac{\tau}{\tau+s}\right|ds.\end{aligned}\tag{4.6}$$

Using estimation, we have

$$|w_2(\tau)-w_1(\tau)| \le M_1K_1[M_1N_1+M_2N_2]+L_1[M_1N_1+M_2N_2],$$

which implies that

$$|w_2(\tau)-w_1(\tau)| \le (M_1K_1+L_1N_1)(M_1N_1+M_2N_2).$$

In same fashion, one can also

$$|x_2(\tau)-x_1(\tau)| \le (M_2K_2+L_2N_2)(M_1N_1+M_2N_2).$$

Now, $n=3$, in (4.4) and (4.5), we have

$$\begin{aligned}|w_3(\tau)-w_2(\tau)| &\le M_1\int_0^1\left|\frac{\tau}{\tau+s}\right|K_1\left[|w_2(s)-w_1(s)|+|x_2(s)-x_1(s)|\right]ds\\ &+L_1\left[|w_2(s)-w_1(s)|+|x_2(s)-x_1(s)|\right]N_1\int_0^1\left|\frac{\tau}{\tau+s}\right|ds\\ &\le M_1K_1\left[(M_1K_1+L_1N_1)(M_1N_1+M_2N_2)+(M_2K_2+L_2N_2)(M_1N_1+M_2N_2)\right]\\ &+L_1N_1\left[(M_1K_1+L_1N_1)(M_1N_1+M_2N_2)+(M_2K_2+L_2N_2)(M_1N_1+M_2N_2)\right],\end{aligned}$$

so,

$$|w_3(\tau)-w_2(\tau)| \le (M_1K_1+L_1N_1)[(M_1K_1+L_1N_1)(M_1N_1+M_2N_2)+(M_2K_2+L_2N_2)(M_1N_1+M_2N_2)].$$

Thus

$$|w_3(\tau)-w_2(\tau)| \le (M_1K_1+L_1N_1)(M_1N_1+M_2N_2)\left[(M_1K_1+L_1N_1)+(M_2K_2+L_2N_2)\right].$$

Similarly

$$|x_3(\tau)-x_2(\tau)| \le (M_1K_1+L_1N_1)(M_1N_1+M_2N_2)\left[(M_1K_1+L_1N_1)+(M_2K_2+L_2N_2)\right].$$

Now, $n=4$, in (4.4) and (4.5), we have

$$\begin{aligned}|w_4(\tau)-w_3(\tau)| &\le M_1K_1\int_0^1\frac{\tau}{\tau+s}\left[|w_3(s)-w_2(s)|+|x_3(s)-x_2(s)|\right]ds\\ &+L_1N_1\left[|w_3(\tau)-w_2(\tau)|+|w_3(\tau)-x_2(\tau)|\right]\int_0^1\frac{\tau}{\tau+s}ds\\ &\le M_1K_1[(M_1K_1+L_1N_1)(M_1N_1+M_2N_2)(M_1K_1+L_1N_1)(M_2N_2+L_2N_2)\\ &+(M_2K_2+L_2N_2)(M_1N_1+M_2N_2)(M_2K_2+L_1N_1)(M_2N_2+L_2N_2)]\\ &+L_1N_1[(M_1K_1+L_1N_1)(M_1N_1+M_2N_2)(M_1K_1+L_1N_1)(M_2K_2+L_2N_2)\\ &+(M_2K_2+L_2N_2)(M_1N_1+M_2N_2)(M_2K_2+L_1N_1)(M_2K_2+L_2N_2)]\end{aligned}$$

$$
\begin{aligned}
|u_4(t)-u_3(t)| \le\, & (M_1K_1+L_1N_1)[(M_1K_1+L_1N_1)(M_1N_1+M_2N_2)(M_1K_1+L_1N_1)(M_2K_2+L_2N_2) \\
& + (M_2K_2+L_2N_2)(M_1N_1+M_2N_2)(M_2K_2+L_1N_1)(M_2K_2+L_2N_2)] \\
& + (M_1K_1+L_1N_1)(M_1N_1+M_2N_2)[(M_1K_1+L_1N_1)(M_2K_2+L_2N_2) \\
& + (M_2K_2+L_2N_2)(M_1N_1+M_2N_2)(M_2K_2+L_1N_1)(M_2K_2+L_2N_2)] < 1.
\end{aligned}
$$

In the same way generalize the procedure and keeping the product sum less than unity, we get that

$$\sum_{n=1}^{\infty} |w_n(\tau)-w_{n-1}(\tau)| \text{ and } \sum_{n=1}^{\infty} |x_n(\tau)-x_{n-1}(\tau)|$$

are convergent. Thus $\{w_n(\tau)\}$ and $\{x_n(\tau)\}$ are uniformly convergent. So

$$
\begin{aligned}
w(\tau) &= h_1(\tau) + \lim_{n\to\infty} g_1(\tau, w_{n-1}(\tau), x_{n-1}(\tau)) \int_0^1 \frac{\tau}{\tau+s} \Phi_1(s, w_{n-1}(s), x_{n-1}(s))ds, \\
x(\tau) &= h_2(\tau) + \lim_{n\to\infty} g_2(\tau, w_{n-1}(\tau), x_{n-1}(\tau)) \int_0^1 \frac{\tau}{\tau+s} \Phi_2(s, w_{n-1}(s), x_{n-1}(s))ds,
\end{aligned}
$$

are convergent sequences. For uniqueness let $(\bar{w}, \bar{x})$ be another solution of (4), then

$$|\bar{w}(\tau)-w_n(\tau)| = \left| g(\tau, \bar{w}(\tau), \bar{x}(\tau)) \int_0^1 \frac{\tau}{\tau+s} \Phi_1(s, \bar{w}(\tau), \bar{x}(\tau))ds - g(\tau, w_n, x_n) \int_0^1 \frac{\tau}{\tau+s} \Phi_1(s, w_n, x_n)ds \right|$$

like (4.1), we can show that $\lim_{n\to\infty} w_n(\tau) = w(\tau) = \bar{w}(\tau)$. Similarly $\lim_{n\to\infty} x_n(\tau) = x(\tau) = \bar{x}(\tau)$. Thus solution (w, x) is unique. □

5. Discussion part

In few papers, Cahndraseker's quadratic integral equations have been considered due to its tremendous applications in applied sciences. But the consideration was limited to simple one. Mostly the researchers considered only scaler class of aforementioned integral equations. Also in some paper coupled system has been considered for existence and uniqueness. In this paper we study the coupled system of the aforesaid integral equations by taking more complicated nonlinearity occurring in the system. Slightly more computation is required to obtain sufficient conditions for existence of solution as well as of uniqueness. We get the required conditions by using Picard's iterative technique which plays an important rules in the construction of the required theory.

6. Example

To demonstrate our main results, we provide the example given bellow as:

Example 6.1. Consider a general Coupled system of Cahndraseker's quadratic integral equations

$$
\begin{cases}
w(\tau) = \tau^2 + \dfrac{\sin(w(\tau)+x(\tau))}{\tau+4} \displaystyle\int_0^1 \frac{\tau}{\tau+s} \frac{\cos(w(\tau)+x(\tau))}{6+\tau^2} ds, \tau \in [0,1], \\
x(\tau) = \tau + \dfrac{\cos(w(\tau)+x(\tau))}{\tau^2+5} \displaystyle\int_0^1 \frac{\tau}{\tau+s} \frac{\sin(w(\tau)+x(\tau))}{6+\tau^2} ds, \tau \in [0,1].
\end{cases}
\tag{6.1}
$$

From above system

$$
\begin{aligned}
& h_1(\tau) = \tau^2, \;\; h_2(\tau) = \tau, \\
& g_1(\tau, w, x) = \sin(w(\tau)+x(\tau)), g_2(\tau, w, x) = \cos(w(\tau)+x(\tau)), \\
& f_1(\tau, w, x) = \cos(w(\tau)+x(\tau)), \Phi_2(\tau, w, x) = \sin(w(\tau)+x(\tau)).
\end{aligned}
$$

Clearly

$$\left|\frac{\sin(w(\tau)+x(\tau))}{\tau+4}\right| \leq \frac{1}{4},\ M_1=\frac{1}{4}, M_2=\frac{1}{5}N_2=N_1=\frac{1}{6}, L_1=\frac{1}{4},\ L_2=\frac{1}{5}K_1=\frac{1}{6}, K_2=\frac{1}{6}.$$

Now computing

$$M_1K_1+L_1N_1+M_2K_2+L_2N_2=\frac{1}{24}+\frac{1}{24}+\frac{1}{30}+\frac{1}{30}=\frac{27}{180}<1.$$

So the coupled system (6.1) has a unique solution.

7. Conclusion

This paper is generalization of [19] where the author obtained the conditions for a coupled system given by

$$w(\tau)=h_1(\tau)+g_1(\tau,x(\tau))\int_0^1\frac{\tau}{\tau+s}\Phi_1(s,x(s))ds, \qquad \tau\in I=[0,1],$$
$$x(\tau)=h_2(\tau)+g_2(\tau,w(\tau))\int_0^1\frac{\tau}{\tau+s}\Phi_2(s,w(s))ds, \qquad \tau\in I=[0,1].$$

While in this paper we extended the above system to the following and obtained the same condition as obtained in [19] using the same technique

$$w(\tau)=h_1(\tau)+g_1(\tau,w(\tau),x(\tau))\int_0^1\frac{\tau}{\tau+s}\Phi_1(s,w(s),x(s))ds, \qquad \tau\in I=[0,1],$$
$$x(\tau)=h_2(\tau)+g_2(\tau,w(\tau),x(\tau))\int_0^1\frac{\tau}{\tau+s}\Phi_2(s,w(s),x(s))ds, \qquad \tau\in I=[0,1].$$

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