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A Note on "Common Coupled Fixed Point Results for Probabilistic φ-Contractions in Menger PGM-Spaces"

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Abstract

In this short note, we point out and rectify an error in a recently published paper "C Zhu, X Mu, Z Wu, *Common coupled fixed point results for probabilistic φ-contractions in Menger PGM-spaces*, J. Nonlinear Sci. Appl., 8 (2015), 1166–1175".

Keywords: Common coupled fixed point, t-norm of H-type, Menger PGM-spaces.
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In [1], the authors showed the existence and uniqueness of common coupled fixed points for probabilistic φ-contractions in the setup of Menger PGM-spaces. The reader should consult [1] for terms not specifically defined in this note.

Remark 1. The authors in [1] claimed that Example 3.1 supports Theorem 2.1. In Theorem 2.1, the t-norm Δ is considered to be a t-norm of H-type such that $\Delta \geq \Delta_p$ but in Example 3.1, the t-norm considered is Δ_p, which is actually not a t-norm of H-type. So, Example 3.1 is not correct.

In order to rectify this mistake, we reconstruct the illustration given as Example 3.1 in [1] as follows:

Example 2. Let $X = [0, +\infty)$ and $\Delta = \Delta_m$, then Δ is a t-norm of H-type such that $\Delta \geq \Delta_p$. Define $G^* : X \times X \times X \to D^+$ by

$$G^*_{x,y,z}(t) = \begin{cases} 0, & t \leq 0, \\ e^{-\max\{|x-y|,|y-z|,|z-x|\}/t}, & t > 0 \end{cases} \tag{1}$$

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for all $x, y, z \in X$. Then (X, G^*, Δ_m) is a complete Menger PGM-space (see, [2]). We define the mappings $T : X \times X \to X$ and $A : X \to X$ by $T(x, y) = 1$ and $A(x) = (2+x)/3$ for all $x, y \in X$, respectively. Clearly $T(X \times X) \subset A(X)$ and A is continuous. Also, A is commutative with T, since for all $x, y \in X$, we have $A(T(x, y)) = A(1) = 1 = T(A(x), A(y))$ and $A(T(y, x)) = A(1) = 1 = T(A(y), A(x))$. Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be a gauge function such that $\varphi^{-1}(\{0\}) = \{0\}$ and $\sum_{n=1}^{\infty} \varphi^n(t) < +\infty$ for any $t > 0$. We verify that for all $x, y, z, p, q, l \in X$ and $t > 0$, the functions A and T satisfy the following inequality (2.1) of [1]:

$$G^*_{T(x,y),T(p,q),T(h,l)}(\varphi(t)) \geq [\Delta(G^*_{Ax,Ap,Ah}(t), G^*_{Ay,Aq,Al}(t))]^{1/2}. \tag{2}$$

For each $x, y, z, p, q, l \in X$ and $t > 0$, we have $G^*_{T(x,y),T(p,q),T(h,l)}(\varphi(t)) = G^*_{1,1,1}(\varphi(t)) = 1$. Then, clearly the inequality (2) holds. Thus all the conditions of Theorem 2.1 of [1] are satisfied. Therefore, by Theorem 2.1 of [1], the mappings A and T have a unique common coupled fixed point in X, which is indeed 1 in the present illustration.

We next give another example in support of Theorem 2.1 in [1].

Example 3. Let $X = [0, +\infty)$ and $\Delta = \Delta_m$, then Δ is a t-norm of H-type such that $\Delta \geq \Delta_p$. Define the mappings $H : [0, +\infty) \to [0, +\infty)$ and $G^* : X \times X \times X \to D^+$ respectively by

$$H(t) = \begin{cases} 0, & t = 0, \\ 1, & t > 0 \end{cases} \tag{3}$$

and

$$G^*_{x,y,z}(t) = \begin{cases} H(t), & x = y = z, \\ \frac{\alpha t}{\alpha t + |x-y| + |y-z| + |z-x|}, & \text{otherwise} \end{cases} \tag{4}$$

for all $x, y, z \in X$, where $\alpha > 0$. Then (X, G^*, Δ_m) is a complete Menger PGM-space (see, [3]). We define the mappings $T : X \times X \to X$ and $A : X \to X$ by $T(x, y) = \beta$ and $A(x) = \frac{3\beta^2+\beta x}{2\beta+2x}$, for all $x, y \in X$ respectively and β is a fixed positive real number. Clearly $T(X \times X) \subset A(X)$, A is continuous and A is commutative with T. Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be a gauge function such that $\varphi^{-1}(\{0\}) = \{0\}$ and $\sum_{n=1}^{\infty} \varphi^n(t) < +\infty$ for any $t > 0$. We verify that for all $x, y, z, p, q, l \in X$ and $t > 0$, the functions A and T satisfy the inequality (2.1) of [1] stated as inequality (2) in the present paper. For each $x, y, z, p, q, l \in X$ and $t > 0$, we have $G^*_{T(x,y),T(p,q),T(h,l)}(\varphi(t)) = G^*_{\beta,\beta,\beta}(\varphi(t)) = 1$. Then, clearly the inequality (2) holds. Thus all the conditions of Theorem 2.1 of [1] are satisfied. Therefore, by applying Theorem 2.1 of [1], β is the unique common coupled fixed point of T and A.

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