Available online at http://cna-journal.com
Commun. Nonlinear Anal. 2 (2020), 75–91
Research Article

Communications in Nonlinear Analysis
Journal Homepage: www.cna-journal.com

Qualitative Theory and Numerical Simulation of SIRC Model Corresponding to Nonlocal Fractional Order derivative

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Abstract

In this article, the existence theory and numerical solutions for fractional order SIRC model via nonlocal fractional order derivative are developed. Using the tools of analysis, the conditions for the existence and stability of the proposed model are established. With the help of Laplace Adomain Decomposition method, we obtain the approximate solutions for the underlying model. In the last part, using Matlab, we plotted various graphs to discuss the proposed model for different fractional order values of ξ.

Keywords: Fractional Derivatives, Fixed point theory, Ulams type Stabilities, Mathematical modeling, Approximate Solutions, Laplace-Adomian decomposition method.
2010 MSC: 34A12, 47H09, 47H30.

1. Introduction

In modern era, several experimental evidences show that natural dynamics follow fractional calculus. Fractional calculus: a fastest growing area of research has applications in diverse and widespread fields of engineering and science such as electromagnetic, viscoelasticity, signal and image processing, quantum mechanics, control theory, non-linear dynamics, biological population models, optimization theory and much more [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Instantly, it is evident that dealing with the dynamical system having memory effects is one of the biggest challenges for researchers. Since the fractional calculus has direct link with the dynamical system (with memory effect). Therefore, fractional differential equations (FDEs): a novel technique is developed to model phenomena related to the dynamics of the aforesaid fields of science [11, 12, 13, 14]. FDEs are global in nature and greater degree of freedom as compared to the conventional differential equations (DEs). Due to this remarkable property, numerous researchers are investigated various features of FDEs concerning the existence, stability analysis and approximate solutions. They utilized

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Received 2020-05-11

different techniques of fixed-point theory and numerical analysis to investigate the existence theory, stability analysis and approximate solution of DFEs refer to [15, 16, 17, 18]. It is important to note that stability analysis and approximate solutions are the key factors of FDEs. Since in various real-world problems either it is quite difficult or it needs too complicated and massive calculations to obtained the exact solution of FDEs. Therefore, in such a situation stability analysis and approximate solutions plays a vital role to tackle the complicated problems involving FDEs. Despite the fact that there are verities of stabilities such as Lyapunov stability, Exponential stability, Asymptotic stability, Mittag-Leffler stability [19, 20, 21, 22], probably the most reliable one is Ullam-Hyers (UH) stability which is the consequent of the correspondence between Ullam [22] and Hyers [23] in 1940-41. The UH stability was further modified and generalized by various other researchers [24, 25, 26].

Like classical derivatives of calculus, fractional calculus also involves various types of fractional derivatives such as Riemann-Liouville (RL), Caputo (C), Hamdard (H), Caputo Febrizo (CF), Atangana-Baleanu (AB) and Atangana-Baleanu-Caputo (ABC).The derivatives in sense of Riemann-Liouville and Caputo are vastly used and well explored by several researchers[27, 28, 29]. Since the classical fractional derivatives involving a singular kernel which could not determine the nonlocal dynamics always. Therefore, the notion of nonsingular derivatives has been introduced. In 2016, Caputo and Fabrizoinitiated nonsingular derivative involving exponential function. In subsequent years the concerned derivative were generalized by Atangana-Baleanu-Caputo and know known as ABC derivative. The operator is recently construed non-local,without singular kernel and reliable differential operator, which are applied in modeling of various real-world phenomena [30]. The complex situations due to singular kernel has replaced by involving exponential and power decay law, for detail see [31, 32]. The problems under ABC derivative have been studied for iterative solutions mostly by using some integral transform, but very rarely investigated from qualitative and numerical aspects.

Laplace transform is an integral transform used in various biological and engineering problems. More precisely, it is an influential tool to solve a verity of FDEs with initial conditions. Also, it is used for the interpretation of time invariant systems such as harmonic oscillation, electric circuit, mechanical systems and optical devices. In addition, it is used to change the problem from time domain to frequency domain. Using Laplace Transform, a differential equation is converted to an algebraic equation which can be solved through algebraic techniques. Moreover, the laplace transform is invertible, the inverse laplace transform take a function of complex variable and yield a function of real variables. A verity of numerical computational techniques like HPM [33], VIM [34], GDM [35], HVIM [36] and ADM etc. Probably, one of the most accurate and efficient approximate technique for the solution of FDE's is Laplace transform coupled with ADM, recognize as Laplace Adomiande composition method (LADM). The said technique is a powerful too use to obtain numerical solutions for wide rang of FDEs also, it provides the solutions of an infinite series in which each term can be determined easily.

In real world situation, either to study accurately the biological behaviors of diseases or to precisely tackle an engineering problem, a powerful mathematical tool which produce more reliable results is known as mathematical modeling. In this regard, various mathematical modeling tools are used to study the transmission and developed a better plane for the prevention of mankind from these deadly infectious diseases, see [37, 38, 39, 40]. It has been observed that proper understanding and implementation for the control strategies against the transmission of spreading diseases in the community is unbreakable challenge for mankind. However, to some extend the aforementioned techniques play a key role to plane, prevent and eliminate the deadly diseases from the community the readers further refer to [41, 42, 43].

The modern world, despite of having precise techniques and sophisticated technologies to tackle various problems of engineering and science, is striving to fight an infectious disease like influenza which is spreading dramatically in the world community. Influenza is one of the dangerous diseases which intensively affected both the developed and underdeveloped countries [49]. Influenza mainly caused by three types of viruses namely type A, B and C [50]. Among these three types of viruses, type A is epidemiologically the most dangerous to human being because it can recombine its genes to produce new subtypes of viruses. Also, the same effect has been seen in the animal population like swine, horses and birds. For further study refer

Parameters	Description
$S(t)$	Susceptible population
$I(t)$	Infectious population
$R(t)$	Recovered population
$C(t)$	Cross-immune
ν	Mortality rate
ϑ	Rate of progression from infective to recovered per year
μ	Rate of progression from recovered to cross-immune per year
η	Rate of progression from recovered to susceptible per year
σ	Recruitment rate of cross-immune into the infective
ϱ	Contact rate per year

Table 1: Parameters used in model (1.1).

to [51, 52]. important one for human being, it can recombine its genes with those of strain circulating in animals' populations, like swine, horses, birds, and so on. Since the surface of Influenza of type A has tiny variation which often referred as dirt. Therefore, the virus can easily attack on the human immune system as a result, it caused severe infection and causes death to mankind. For prevention, a person needs vaccination. On the other hand, a genetic mutation take place in influenza type A which causes new subtypes of viruses. As a result, it can lead to outbreaking a global pandemic influenza, for example Spanish outbroke H!N! influenza virus in 1918-1919 that took about 20-40 million lives. In addition, in 1957-1958, Asia influenza blow up due to the result of H2N2 virus. It is important to note that an influenza not only effect clinically ill but also took the lives of healthy individuals which caused economic loss for many developed countries as well. In America, about 10-15 billion dollars economic loss was indicated reported, further see [56]. In such a situation, it is essential to study accurately the biological behaviors of diseases and to planned a precise model for the prevention of the infectious disease like influenza. Therefore, a powerful mathematical tool which produce more reliable results is known as mathematical modeling. In the last few decades, a verity of epidemic models has been proposed to tackle the aforesaid similar problems. The said models are predicting the spread of influenza in human population based on classical susceptible-infected-removed (SIR) model developed by Kermack and McKendrick ([55]). Beside this, Casagrandi et al. [53]) developed the SIRC model by introducing new component C, which is called cross-immune compartment, to the SIR model. The intermediate state between the fully protected (R) and fully susceptible (S) in describes by this cross-immune (C). The authors also presented numerically the dynamical behavior of this model in [54].

In modern era, the study of such infectious diseases is still a central focus for the researchers. In this regard, we predict and investigate the dynamics of fractional order SIRC model (1.1) via ABC fractional operator. We develop a precise mechanism how to prevent the transmission of infectious disease in the community. The capture fractional order SIRC under Atangaba-Baleau-Caputo derivative is given as:

$$\begin{cases} {}_0^{ABC}D_t^{\xi}S(t) = \nu(1-S(t)) - \varrho S(t)I(t) + \eta C(t), \\ {}_0^{ABC}D_t^{\xi}I(t) = \varrho S(t)I(t) + \sigma\varrho C(t)I(t) - (\nu+\vartheta)I(t), \\ {}_0^{ABC}D_t^{\xi}R(t) = (1-\sigma)\varrho C(t)I(t) + \vartheta I - (\nu+\mu)R(t), \\ {}_0^{ABC}D_t^{\xi}C(t) = \mu R(t) - \varrho C(t)I(t) - (\nu+\eta)C(t). \end{cases} \tag{1.1}$$

With initial conditions $S(0) = s_0$, $I(0) = i_0$, $R(0) = r_0$ and $C(0) = c_0$, where $0 < \xi \leq 1$. The parameters involved in (1.1) and their physical interpretation is expressed in table(1). Here we also assume that all the parameters are non-negative. Corresponding to model (1.1), we use fixed point approach to investigate some results that ensure the existence of proposed model and its solution. We use Banach and Schauder's theorems from fixed point theory. We obtain the estimated solution of concerned model of non-integer order via Laplace transform combined with Adomian decomposition method. To justified the results obtained by aforementioned procedure, we use Mapple-13 and assigned different values to the parameters and supplement conditions.

An efficient techniques by which we can find both explicit and analytic solutions for the system of equations rate of change, was initiated by Adomain is known as LADM, in 1980. The aforesaid techniques has an efficient techniques, which works outstandingly in both cases that is boundary and initial value problems. The consider techniques also works accurately in a system of stochastic differential equations. LADM does not needs liberalization or perturbation, like other existing computational and analytical schemes, that needs for exploring the dynamical behavior of complex dynamical systems. The committed techniques provides extensive results for the solutions of FODEs and as well as for analytical solution for the verity problem of nonlinear equations. In this paper, we utilized techniques of Adomain polynomial to decomposed the non-linearity and Laplace to convert the deserts problem to the form algebraic equations, see[44]. Recently, the proposed techniques are used to deal with nonsingular FODEs, to obtained very fruitful results, (see [45]). Furthermore, we remark that the obtained results via the considered method is in a form of convergent series, that converges to the exact results uniformly. Thanks to the results of analysis [46, 47, 48], one can easily prove the convergent of the proposed method.

2. Preliminaries

Definition 2.1. If $\Psi(t) \in \mathbb{H}^1(0, \mathcal{T})$ and $\xi \in (0, 1]$, then the $\mathcal{ABC}$ derivative is defined as

$$^{ABC}D^{\xi}_{+0}\Psi(t) = \frac{ABC(\xi)}{1-\xi}\int_0^t \frac{d}{dx}\Psi(y)M_{\xi}\left[\frac{-\xi}{1-\xi}(t-y)\right]dy, \tag{2.1}$$

if we replace $M_{\xi}\left[\frac{-\xi}{1-\xi}(t-y)\right]dy$ by $M_1 = exp\left[\frac{-\xi}{1-\xi}(t-y)\right]$, then one obtain derivative is known as Caputo-Fabrizo. Also, we have

$$^{ABC}D^{\xi}_{+0}[Constant] = 0.$$

Where $ABC(\xi)$ is known as normalization function which is defined as $ABC(0) = ABC(1) = 1$. M_{ξ} stands for famous function known as Mittag-Leffler, the generalization of exponential function *([30],[31], [32])*.

Definition 2.2. If $z \in L[0, T]$, then the fractional integral defined in sense of ABC as

$$^{ABC}D^{\xi}_{+0}z(t) = \frac{1-\xi}{ABC(\xi)}z(t) + \frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t (t-y)^{\xi-1}z(y)dy. \tag{2.2}$$

Lemma 2.3. *[27] solution of the problem for* $1 > \xi > 0$

$$^{ABC}D^{\xi}_{+0}U(t) = x(t), \ \ t \in [0, T],$$
$$U(0) = U_0$$

is given by

$$U(t) = U_0 + \frac{(1-\xi)}{ABC(\xi)}x(t) + \frac{\xi}{\Gamma(\xi)ABC(\xi)}\int_0^t (t-y)^{\xi-1}x(y)dy.$$

Definition 2.4. Laplace transform for ABC derivative of function $\phi(t)$ is given by

$$\mathscr{L}[^{ABC}D^{\xi}_{0}\phi(t)] = \frac{ABC(\xi)}{s^{\xi}(1-\xi)+\xi}\left[s^{\xi}\mathscr{L}[\phi(t)] - s^{\xi-1}\phi(0)\right].$$

Key point: For qualitative analysis, we define Banach space $\mathbb{Z} = \mathbb{X}\times\mathbb{X}\times\mathbb{X}\times\mathbb{X}$, with $\mathbb{X} = C[0, T]$ under the norm defined by $||M|| = ||(S, I, R, C)|| = max_t \in [0, T]||S(t) + I(t) + R(t) + C(t)||$. For our main result, the following theorem will be used.

Theorem 2.5. *Let $\mathbb{B}$ be a convex subset of $\mathbb{Z}$, assuming that the operators $\mathbb{F}, \mathbb{G}$ with (1). $\mathbb{F}u + \mathbb{G}u \in \mathbb{B}$ for each $u \in \mathbb{B}$.*
(2). $\mathbb{F}$ is contraction.
(3). $\mathbb{G}$ is continuous and compact.
Then $\mathbb{F}u + \mathbb{G}u = u$, has at least one solution.

3. Qualitative Theory

The concerned section, is dedicated to the existence and uniqueness of the solution of BVP of FDEs. FDEs provide powerful tools, that describes different physical, biological and dynamical phenomenon in mathematical concepts. In last two decades, due to the versatile applications of FDEs, the researchers give more attention to the existence of solutions for FDEs. Another important aspects of FDEs, that it is widely used in the different fields of applied science and technology is devoted to the stability analysis. In this section we determined existence result for the proposed model (1.1), using fixed point theorem due to Banach type for the existence and uniqueness of solution. In this regard, we first define the following function

$$\begin{cases} \Upsilon_1(t, S, I, R, C) = \nu(1 - S) - \varrho SI + \eta C, \\ \Upsilon_2(t, S, I, R, C) = \varrho SI + \sigma\varrho CI - (\nu + \vartheta)I, \\ \Upsilon_3(t, S, I, R, C) = (1 - \sigma)\varrho CI + \vartheta I - (\nu + \mu)R, \\ \Upsilon_4(t, S, I, R, C) = \mu R - \varrho CI - (\nu + \eta)C. \end{cases} \tag{3.1}$$

With the help of (3.1), the constructed system is written in the following form

$$\begin{aligned} {}^{ABC}D^{\xi}_{+0}U(t) &= \Upsilon(t, U(t)),\ t \in [0, T],\ 0 < \xi \le 1, \\ U(0) &= U_0. \end{aligned} \tag{3.2}$$

Using Lemma (2.3), equation (3.2) becomes

$$U(t) = U_0(t) + \Big[\Upsilon((t, U(t)) - \Upsilon_0(t)\Big]\frac{1-\xi}{ABC(\xi)} + \frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t (t-y)^{\xi-1}\Upsilon(y, U(y))dy, \quad for\ \ 0 \le y \le t \le 1, \tag{3.3}$$

where

$$U(t) = \begin{cases} S(t) \\ I(t) \\ R(t) \\ C(t) \end{cases}, U_0(t) = \begin{cases} S_0 \\ I_0 \\ R_0 \\ C_0 \end{cases}, \Upsilon(t, U(t)) = \begin{cases} g_1(t, S, I, R, C) \\ g_2(t, S, I, R, C) \\ g_3(t, S, I, R, C) \\ g_4(t, S, I, R, C) \end{cases}, \Upsilon_0(t) \begin{cases} g_1(0, S_0, I_0, R_0, C_0) \\ g_2(0, S_0, I_0, R_0, C_0) \\ g_3(0, S_0, I_0, R_0, C_0) \\ g_4(0, S_0, I_0, R_0, C_0) \end{cases}. \tag{3.4}$$

Using (3.3) and (3.4), define two operators $\mathbb{F}$ and $\mathbb{G}$, using (3.3)

$$\begin{aligned} \mathbb{F}u &= U_0(t) + \Big[\Upsilon(t, U(t)) - \Upsilon_0(t)\Big]\frac{1-\xi}{ABC(\xi)}, \\ \mathbb{G}u &= \frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t (t-y)^{\xi-1}\Upsilon(y, U(y))dy. \end{aligned} \tag{3.5}$$

For growth condition, Lipschitizian assumptions, existence and uniqueness, the following holds
(L_1) There exists constants b^* and c^*, such that

$$|\Upsilon(t, U(t))| \le b^*|U(t)| + c^*.$$

(L_2)There exists constant $K_p > 0$, for every $u, \bar{u} \in \mathbb{X}$, such that

$$|\Upsilon(t, U(t)) - \Upsilon(t, \bar{U}(t))| \le K_p||u - \bar{u}||.$$

Theorem 3.1. *If* (L_1) *and* (L_2) *holds, then equation* (3.3) *has at least one solution which means that the consider system* (1.1) *has one solution if*

$$\frac{(1-\xi)K_p}{ABC(\xi)} < 1.$$

***Proof*.** To show that $\mathbb{F}$ is contraction, let $\bar{u} \in \mathbb{B}$, where $\mathbb{B} = \{u \in \mathbb{Z} : ||u|| \leq r, r > 0\}$ is closed convex set. Using the definition of $\mathbb{F}$ from (3.5), we get

$$\begin{aligned} ||\mathbb{F}u - \mathbb{F}\bar{u}|| &= \frac{(1-\xi)}{ABC(\xi)} max_{t\in[0,T]} \Big| \Upsilon(t, U(t)) - \Upsilon(t, \bar{U}(t)) \Big|, \\ &\leq \frac{(1-\xi)_p}{ABC(\xi)} ||u - \bar{u}||. \end{aligned} \tag{3.6}$$

Hence $\mathbb{F}$ is contraction.
To show that $\mathbb{G}$ is relatively compact, we have to show that $\mathbb{G}$ is bounded, and continuous. For this, we proceeds as follow:
It is obvious that $\mathbb{G}$ is continuous as Υ is continuous, also for $u \in \mathbb{B}$, we have

$$\begin{aligned} |\mathbb{G}(u)| &= \max_{t\in[0,T]} \frac{\xi}{ABC(\xi)\Gamma(\xi)} \Bigg|\Bigg| \int_0^t (t-y)^{\xi-1} \Upsilon(y, U(y)) dy \Bigg|\Bigg|, \\ &\leq \frac{\xi}{ABC(\xi)\Gamma(\xi)} \int_0^T (T-y)^{\xi-1} |\Upsilon(y, U(y))| dy, \\ &\leq \frac{\xi T^\xi}{ABC(\xi)\Gamma(\xi)} [b^* r + c^*]. \end{aligned} \tag{3.7}$$

Hence (3.7) shows that $\mathbb{G}$ is bounded, for equi-continuous, let $t_1 > t_2 \in [0, T]$, such that

$$\begin{aligned} |\mathbb{G}U(t_1) - \mathbb{G}U(t_2)| &= \frac{\xi}{ABC(\xi)\Gamma(\xi)} \Bigg| \int_0^{t_1} (t_1 - y)^{\xi-1} \Upsilon(y), U(y) dy - \int_0^{t_2} (t_2 - y)^{\xi-1} \Upsilon(y, U(y)) dy \Bigg|, \\ &\leq \frac{[b^* r + c^*]}{ABC(\xi)\Gamma(\xi)} [t_1^\xi - t_2^\xi]. \end{aligned} \tag{3.8}$$

As $t_1 \to t_2$, right hand side of (3.8) tends to zero, also $\mathbb{G}$ is continuous and so

$$|\mathbb{G}U(t_1) - \mathbb{G}U(t_2)| \to 0, \ as \ t_1 \to t_2.$$

Hence $\mathbb{G}$ is bounded and continuous , therefore $\mathbb{G}$ is uniformly continuous and bounded. By Arzela'-Ascoli theorem $\mathbb{G}$ is relatively compact and so completely continuous. Using theorem (3.1), the integral equation (3.3) has atleast one solution and therefore, the system has atleast one solution.
For uniqueness we provide the following result.

Theorem 3.2. *Under assumption* (L_2), *the integral equation* (3.3) *has unique solution which shows that consider system* (1.1) *has the unique result if*

$$\left[\frac{(1-\xi)K_p}{ABC(\xi)} + \frac{\xi T^\xi K_p}{ABC(\xi)\Gamma(\xi)}\right] < 1.$$

***Proof*.** Let define $\mathbb{T} : \mathbb{Z} \to \mathbb{Z}$ by

$$\mathbb{T}U(t) = U_0(t) + \Big[\Upsilon(t, U(t)) - \Upsilon_0(t)\Big] \frac{1-\xi}{ABC(\xi)} + \frac{\xi}{ABC(\xi)\Gamma(\xi)} \int_0^t (t-y)^{\xi-1} \Upsilon(y, U(y)) dy, \ t \in [0, T]. \tag{3.9}$$

Let $u, \bar{u} \in \mathbb{Z}$, then

$$
\begin{aligned}
||\mathbb{T}u - \mathbb{T}\bar{u}|| \leq & \frac{(1-\xi)}{ABC(\Gamma(\xi))} max_{t\in[0,T]}\Big|\Upsilon(t,U(t)) - \Upsilon(t,\bar{U}(t))\Big|, \\
& + \frac{\xi}{ABC(\xi)\Gamma(\xi)} max_{t\in[0,T]}\Big|\int_0^t (t-y)^{\xi-1}\Upsilon(y,U(y))dy - \int_0^t (t-y)^{\xi-1}\Upsilon(y,\bar{U}(y))dy\Big|, \\
& \leq \left[\frac{(1-\xi)K_p}{ABC(\xi)} + \frac{\xi T^\xi K_p}{ABC(\xi)\Gamma(\xi)}\right]||u-\bar{u}||, \\
& \leq \Omega||u-\bar{u}||,
\end{aligned}
\tag{3.10}
$$

where

$$
\Omega = \left[\frac{(1-\xi)K_p}{ABC(\xi)} + \frac{\xi T^\xi K_p}{ABC(\xi)\Gamma(\xi)}\right]. \tag{3.11}
$$

From (3.10), $\mathbb{T}$ in contraction. Therefor, the integral equation (3.3) has a unique solution. Thus system (1.1) has a unique solution.

4. STABILITY ANALYSIS

For the stability of the considered problem, we consider a small peturbation $\alpha \in C[0,T]$, which depends on the solution only and $\alpha(0) = 0$. Next
(i) $|\alpha(t)| \leq \epsilon, for \ \ \epsilon > 0$
(ii) ${}^{ABC}D^{\xi}_{+0}(U(t)) = \Upsilon(t,U(t)) + \alpha(t), \forall \ t \in [0,T]$.

Lemma 4.1. *Solution of the perturb problem*

$$
\begin{cases}
{}_{0}^{ABC}D^{\xi}_{+0}U(t) = \Psi(t,U(t)) + \alpha(t), \\
U(0) = U_0,
\end{cases}
\tag{4.1}
$$

satisfying the following relation

$$
\begin{aligned}
&\left|U(t) - \left(U_0(t) + \Big[\Upsilon(t,U(t)) - \Upsilon_0(t)\Big]\frac{1-\xi}{ABC(\xi)} + \frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t (t-y)^{\xi-1}\Upsilon(y,U(y))dy\right)\right|, \\
&\leq \Xi_{T,\xi}\epsilon,
\end{aligned}
\tag{4.2}
$$

where

$$
\Xi_{T,\xi} = \frac{\Gamma(\xi)(1-\xi) + T^\xi}{ABC(\xi)\Gamma(\xi)}.
$$

Proof. This proof is simple so we omit it.

Theorem 4.2. *Under assumption* (L_2) *and result* (4.2) *in Lemma* (4.1)*, the solution of the concern integral equation* (3.3) *is Ulam-Hyers stable and consequently, the analytical results of the concern system are Ulams-Hyers stable if* $\Omega < 1$.

Proof. Let $\bar{u} \in \mathbb{Z}$ be a unique solution and u $\in \mathbb{Z}$ be any solution of (3.3), then

$$\begin{aligned}|U(t)-\bar{U}(t)| &= \left|U(t)-\left(U_0(t)+\left[\Upsilon(t,\bar{U}(t))-\Upsilon_0(t)\right]\frac{1-\xi}{ABC(\xi)}+\frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t(t-y)^{\xi-1}\Upsilon(y,\bar{U}(y))dy\right)\right|,\\ &\leq \left|U(t)-\left(U_0(t)+\left[\Upsilon(t,U(t))-\Upsilon_0(t)\right]\frac{1-\xi}{ABC(\xi)}+\frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t(t-y)^{\xi-1}\Upsilon(y,U(y))dy\right)\right|\\ &+\left|\left(U_0(t)+\left[\Upsilon(t,U(t))-\Upsilon_0(t)\right]\frac{1-\xi}{ABC(\xi)}+\frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t(t-y)^{\xi-1}\Upsilon(y,U(y))dy\right)\right.\\ &\left.-\left(U_0(t)+\left[\Upsilon(t,\bar{U}(t))-\Upsilon_0(t)\right]\frac{1-\xi}{ABC(\xi)}+\frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t(t-y)^{\xi-1}\Upsilon(y,\bar{U}(y))dy\right)\right|,\\ &\leq \Xi_{T,\xi}.\epsilon+\frac{(1-\xi)K_p}{ABC(\xi)}||u-\bar{u}||+\frac{\xi T^{\xi}K_p}{ABC(\xi)\Gamma(\xi)}||u-\bar{u}||\\ &\leq \Xi_{T,\xi}.\epsilon+\Omega||u-\bar{u}||.\end{aligned} \tag{4.3}$$

From (4.3), we can write

$$||U-\bar{U}|| \leq \frac{\Xi_{T,\xi}\epsilon}{1-\Omega}. \tag{4.4}$$

From (4.4), we concluded that the solution of (3.3) is Ullam-Hyers stable and consequently generalized Ulam-Hyers Stable by using $\Upsilon_U(\epsilon) = \Xi_{T,\xi}\epsilon, \Upsilon_U(0) = 0$, which shows that the solution of the proposed problem is Ulam-Hyers stable and also generalized Ulam-Hyers stable.
Let us consider the following suppositions
(i) $|\alpha(t)| \leq \phi(t)\epsilon, for\ \ \epsilon > 0$
(ii) ${}^{ABC}D^{\xi}_{+0}(U(t)) = \Upsilon(t,U(t)) + \alpha(t), \forall\ \ t \in [0,T]$.

Lemma 4.3. *The following holds for* (4.1)

$$\begin{aligned}&\left|U(t)-\left(U_0(t)+\left[\Upsilon(t,U(t))-\Upsilon_0(t)\right]\frac{1-\xi}{ABC(\xi)}+\frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t(t-y)^{\xi-1}\Upsilon(y,U(y))dy\right)\right|\\ &\leq \Xi_{T,\xi}\phi(t)\epsilon.\end{aligned} \tag{4.5}$$

Proof. We can easily get the required result, so we omit it.

Theorem 4.4. *Under the Lemma* (4.3)*, the solution of the consider problem is Ulam-Hyers-Rassias stable and consequently generalized Ulam-Hyers-Rassias stable.*

Proof. Let $\bar{u} \in \mathbb{Z}$ be a unique solution and u $\in \mathbb{Z}$ be any solution of (3.3), then

$$\begin{aligned}|U(t)-\bar{U}(t)| &= \left|U(t)-\left(U_0(t)+\left[\Upsilon(t,\bar{U}(t))-\Upsilon_0(t)\right]\frac{1-\xi}{ABC(\xi)}+\frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t(t-y)^{\xi-1}\Upsilon(y,\bar{U}(y))dy\right)\right|,\\ &\leq \left|U(t)-\left(U_0(t)+\left[\Upsilon(t,U(t))-\Upsilon_0(t)\right]\frac{1-\xi}{ABC(\xi)}+\frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t(t-y)^{\xi-1}\Upsilon(y,U(y))dy\right)\right|\\ &+\left|\left(U_0(t)+\left[\Upsilon(t,U(t))-\Upsilon_0(t)\right]\frac{1-\xi}{ABC(\xi)}+\frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t(t-y)^{\xi-1}\Upsilon(y,U(y))dy\right)\right.\\ &\left.-\left(U_0(t)+\left[\Upsilon(t,\bar{U}(t))-\Upsilon_0(t)\right]\frac{1-\xi}{ABC(\xi)}+\frac{\xi}{ABC(\xi)\Gamma(\xi)}\int_0^t(t-y)^{\xi-1}\Upsilon(y,\bar{U}(y))dy\right)\right|,\\ &\leq \Xi_{T,\xi}\phi(t)+\frac{(1-\xi)K_p}{ABC(\xi)}||u-\bar{u}||+\frac{\xi T^{\xi}K_p}{ABC(\xi)\Gamma(\xi)}||u-\bar{u}||,\\ &\leq \Xi_{T,\xi}\phi(t)\epsilon+\Omega||u-\bar{u}||,\end{aligned} \tag{4.6}$$

we can write, from (4.6)

$$||U-\bar{U}|| \leq \frac{\Xi_{T,\xi}\phi(t)\epsilon}{1-\Omega}. \tag{4.7}$$

Hence the solution of (3.3) is Ulam-Hyers-Rassias stable and consequently generalized Ulam-Hyers-Rassias stable.

5. General procedure for approximate solution

In this segment of the article, we developed the approximate scheme of the proposed model (1.1). Taking Laplace transform of (1.1), we have

$$\begin{cases} {}^{ABC}D^{\xi}_{+0}S(t) = \nu(1-S(t)) - \varrho S(t)I(t) + \eta C(t), \\ {}^{ABC}D^{\xi}_{+0}I(t) = \varrho S(t)I(t) + \sigma\varrho C(t)I(t) - (\nu+\vartheta)I(t), \\ {}^{ABC}D^{\xi}_{+0}R(t) = (1-\sigma)\varrho C(t)I(t) + \vartheta I - (\nu+\mu)R(t), \\ {}^{ABC}D^{\xi}_{+0}C(t) = \mu R(t) - \varrho C(t)I(t) - (\nu+\eta)C(t), \end{cases} \tag{5.1}$$

Applying Laplace transform in the sense of ABC fractional derivative on (5.1), we have

$$\begin{cases} \mathscr{L}\{S(t)\} = \dfrac{S(0)}{s} + \left\{\dfrac{s^{\xi}(1-\xi)+\xi}{s^{\xi}ABC(\xi)}\right\}\mathscr{L}\{\nu(1-S(t)) - \varrho S(t)I(t) + \eta C(t)\}, \\ \mathscr{L}\{I(t)\} = \dfrac{I(0)}{s} + \left\{\dfrac{s^{\xi}(1-\xi)+\xi}{s^{\xi}ABC(\xi)}\right\}\mathscr{L}\{\varrho S(t)I(t) + \sigma\varrho C(t)I(t) - (\nu+\vartheta)I(t)\}, \\ \mathscr{L}\{R(t)\} = \dfrac{R(0)}{s} + \left\{\dfrac{s^{\xi}(1-\xi)+\xi}{s^{\xi}ABC(\xi)}\right\}\mathscr{L}\{(1-\sigma)\varrho C(t)I(t) + \vartheta I(t) - (\nu+\mu)R(t)\}, \\ \mathscr{L}\{C(t)\} = \dfrac{c(0)}{s} + \left\{\dfrac{s^{\xi}(1-\xi)+\xi}{s^{\xi}ABC(\xi)}\right\}\mathscr{L}\{\mu R(t) - \varrho C(t)I(t) - (\nu+\eta)C(t)\}. \end{cases} \tag{5.2}$$

Applying inverse Laplace and using the initial conditions on (5.2), we have

$$\begin{cases} S(t) = s_0 + \mathscr{L}^{-1}\left[\left\{\dfrac{s^{\xi}(1-\xi)+\xi}{s^{\xi}ABC(\xi)}\right\}\mathscr{L}\{\nu(1-S(t)) - \varrho S(t)I(t) + \eta C(t)\}\right], \\ I(t) = i_0 + \mathscr{L}^{-1}\left[\left\{\dfrac{s^{\xi}(1-\xi)+\xi}{s^{\xi}ABC(\xi)}\right\}\mathscr{L}\{\varrho S(t)I(t) + \sigma\varrho C(t)I(t) - (\nu+\vartheta)I(t)\}\right], \\ R(t) = r_0 + \mathscr{L}^{-1}\left[\left\{\dfrac{s^{\xi}(1-\xi)+\xi}{s^{\xi}ABC(\xi)}\right\}\mathscr{L}\{(1-\sigma)\varrho C(t)I(t) + \vartheta I(t) - (\nu+\mu)R(t)\}\right], \\ C(t) = c_0 + \mathscr{L}^{-1}\left[\left\{\dfrac{s^{\xi}(1-\xi)+\xi}{s^{\xi}ABC(\xi)}\right\}\mathscr{L}\{\mu R(t) - \varrho C(t)I(t) - (\nu+\eta)C(t)\}\right]. \end{cases} \tag{5.3}$$

Let the solution S(t), I(t), R(t) and C(t) is defined in term of series as follow

$$S(t) = \sum_{n=0}^{\infty} S_n(t), \;\; I(t) = \sum_{n=0}^{\infty} I_n(t), \;\; R(t) = \sum_{n=0}^{\infty} R_n(t), \;\; C(t) = \sum_{n=0}^{\infty} C_n(t). \tag{5.4}$$

The given system contains non-linear terms S(t). I(t) and C(t). I(t), which can be decompose in term of LADM, as

$$S(t).I(t) = \sum_{n=0}^{\infty} A_m, \;\; C(t).I(t) = \sum_{n=0}^{\infty} B_m. \tag{5.5}$$

Where A_m and B_m are Adomian's polynomials, defined as

$$A_m = \frac{1}{m!}.\frac{d^m}{d\sigma^m}\left[\sum_{l=0}^{m}\sigma^l S_l.\sum_{l=0}^{m}\sigma^l I_l\right]\Big|_{\sigma=0},$$
$$B_m = \frac{1}{m!}.\frac{d^m}{d\sigma^m}\left[\sum_{l=0}^{m}\sigma^l C_l.\sum_{l=0}^{m}\sigma^l I_l\right]\Big|_{\sigma=0}.$$

Now using (5.4) and (5.5) in (5.2), we have

$$\begin{cases}
\mathscr{L}\{S_0\} = \frac{s_0}{s}, \mathscr{L}\{I_0\} = \frac{I_0}{s}, \mathscr{L}\{R_0\} = \frac{R_0}{s}, \mathscr{L}\{C_0\} = \frac{I_0}{s},\\
\mathscr{L}\{S_1\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\nu - \nu S_0 - \varrho A_0 + \eta C_0\},\\
\mathscr{L}\{I_1\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\varrho A_0 - \sigma\varrho B_0 - (\nu+\vartheta)I_0\},\\
\mathscr{L}\{R_1\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\varrho(1-\sigma)B_0 + \vartheta I_0 - (\nu+\mu)R_0\},\\
\mathscr{L}\{C_1\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\mu R_0 - \varrho B_0 - (\nu+\eta)C_0\},\\
\mathscr{L}\{S_2\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\nu - \nu S_1 - \varrho A_1 + \eta C_1\},\\
\mathscr{L}\{I_2\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\varrho A_1 - \sigma\varrho B_1 - (\nu+\vartheta)I_1\},\\
\mathscr{L}\{R_2\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\varrho(1-\sigma)B_1 + \vartheta I_1 - (\nu+\mu)R_1\},\\
\mathscr{L}\{C_2\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\mu R_1 - \varrho B_1 - (\nu+\eta)C_1\},\\
.\\
.\\
.\\
\mathscr{L}\{S_n\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\nu - \nu S_{n-1} - \varrho A_{n-1} + \eta C_{n-1}\},\\
\mathscr{L}\{I_n\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\varrho A_{n-1} - \sigma\varrho B_{n-1} - (\nu+\vartheta)I_{n-1}\},\\
\mathscr{L}\{R_n\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\varrho(1-\sigma)B_{n-1} + \vartheta I_{n-1} - (\nu+\mu)R_{n-1}\},\\
\mathscr{L}\{C_n\} = \left[\frac{s^\xi(1-\xi)+\xi}{s^\xi ABC(\xi)}\right].\mathscr{L}\{\mu R_{n-1} - \varrho B_{n-1} - (\nu+\eta)C_{n-1}\},
\end{cases} \tag{5.6}$$

applying inverse Laplace on both side of (5.6), we get

$$
\begin{cases}
S_0(t) = S_0, I_0(t) = I_0, R_0(t) = R_0, C_0(t) = C_0, \\
S_1 = \frac{2-\xi}{2}\left\{(1-\xi) + \frac{\xi}{\xi!}t^{\xi}\right\}\{\nu - \nu S_0 - \varrho A_0 + \eta C_0\}, \\
I_1 = \frac{2-\xi}{2}\left\{(1-\xi) + \frac{\xi}{\xi!}t^{\xi}\right\}\{\varrho A_0 - \sigma\varrho B_0 - (\nu+\vartheta)I_0\}, \\
R_1 = \frac{2-\xi}{2}\left\{(1-\xi) + \frac{\xi}{\xi!}t^{\xi}\right\}\{\varrho(1-\sigma)B_0 + \vartheta I_0 - (\nu+\mu)R_0\}, \\
C_1 = \frac{2-\xi}{2}\left\{(1-\xi) + \frac{\xi}{\xi!}t^{\xi}\right\}\{\mu R_0 - \varrho B_0 - (\nu+\eta)C_0\}, \\
S_2 = \nu\left(\frac{2-\xi}{2}\right)\left((1-\xi) + \frac{\xi}{\xi!}t^{\xi}\right) - \left[\frac{(2-\xi)^2}{4}\right]\left[(1-\xi)^2 + \frac{\xi^2}{2\xi!}t^{2\xi} + \frac{2\xi(1-\xi)}{\xi!}t^{\xi}\right]\Big[\nu(\nu - \nu S_0 - \varrho S_0 I_0 + \eta C_0) \\
\quad (\nu + \varrho I_0) + \varrho S_0\{\varrho S_0 I_0 - \sigma\varrho C_0 I_0 - (\nu+\vartheta)I_0\} - \eta\{\mu R_0 - \varrho C_0 I_0 - (\nu+\eta\}C_0)\Big], \\
I_2 = \left(\frac{(2-\xi)^2}{4}\right)\left[(1-\xi)^2 + \frac{\xi^2}{2\xi!}t^{2\xi} + \frac{2\xi(1-\xi)}{\xi!}t^{\xi}\right]\Big[\{\varrho A_0 - \sigma\varrho C_0 I_0 - (\nu+\vartheta)I_0\}\{\varrho S_0 - \sigma\varrho C_0 - (\nu+\vartheta)\} \\
\quad - \varrho I_0\{\nu - \nu S_0 - \varrho S_0 I_0 + \eta C_0\} + \sigma\varrho I_0\{\mu R_o - \varrho C_0 I_0 - (\nu+\eta)C_0\}\Big], \\
R_2 = \left(\frac{(2-\xi)^2}{4}\right)\left[(1-\xi)^2 + \frac{\xi^2}{2\xi!}t^{2\xi} + \frac{2\xi(1-\xi)}{\xi!}t^{\xi}\right]\Big[\{\varrho S_0 I_0 - \sigma\varrho C_0 I_0 - (\nu+\vartheta)I_0\}\{\varrho(1-\sigma)C_0 + \vartheta\} \\
\quad + \varrho(1-\sigma)I_0\{\mu R_0 - \varrho C_0 I_0 - (\nu+\eta)C_0\} - (\nu+\mu)\{\varrho(1-\sigma)C_0 I_0 + \vartheta I_0 - (\nu+\mu)R_0\}\Big], \\
C_2 = \left(\frac{(2-\xi)^2}{4}\right)\left[(1-\xi)^2 + \frac{\xi^2}{2\xi!}t^{2\xi} + \frac{\xi(1-\xi)}{2\xi!}t^{\xi}\right]\Big[\mu\{\varrho(1-\sigma)C_0 I_0 + \vartheta I_0 - (\nu+\mu)R_0\} \\
\quad - \varrho C_0\{\varrho S_0 I_0 - \sigma\varrho C_0 I_0 - (\nu+\vartheta)I_0\} - \{\mu R_0 - \varrho C_0 I_0 - (\nu+\eta)C_0\}\{\varrho I_0 + (\nu+\eta)\}\Big],
\end{cases}
\tag{5.7}
$$

By doing the same process as above, we can obtain other terms. The solution in term of infinite series up-to three terms is given by

$$\begin{cases} S_n = S_0 + \frac{2-\xi}{2}\left\{(1-\xi) + \frac{\xi}{\xi!}t^{\xi}\right\}\{\nu - \nu S_0 - \varrho A_0 + \eta C_0\} + \\ \quad + \nu\left(\frac{2-\xi}{2}\right)\left((1-\xi) + \frac{\xi}{\xi!}t^{\xi}\right) - \left[\frac{(2-\xi)^2}{4}\right]\left[(1-\xi)^2 + \frac{\xi^2}{2\xi!}t^{2\xi} + \frac{2\xi(1-\xi)}{\xi!}t^{\xi}\right]\Big[\nu(\nu - \nu S_0 - \varrho S_0 I_0 + \eta C_0) \\ \quad (\nu + \varrho I_0) + \varrho S_0\{\varrho S_0 I_0 - \sigma\varrho C_0 I_0 - (\nu+\vartheta)I_0\} - \eta\{\mu R_0 - \varrho C_0 I_0 - (\nu+\eta\}C_0)\Big] + ..., \\ I_n = I_0 + \frac{2-\xi}{2}\left\{(1-\xi) + \frac{\xi}{\xi!}t^{\xi}\right\}\{\varrho A_0 - \sigma\varrho B_0 - (\nu+\vartheta)I_0\} \\ \quad + \left(\frac{(2-\xi)^2}{4}\right)\left[(1-\xi)^2 + \frac{\xi^2}{2\xi!}t^{2\xi} + \frac{2\xi(1-\xi)}{\xi!}t^{\xi}\right]\Big[\{\varrho A_0 - \sigma\varrho C_0 I_0 - (\nu+\vartheta)I_0\}\{\varrho S_0 - \sigma\varrho C_0 - (\nu+\vartheta)\} \\ \quad - \varrho I_0\{\nu - \nu S_0 - \varrho S_0 I_0 + \eta C_0\} + \sigma\varrho I_0\{\mu R_o - \varrho C_0 I_0 - (\nu+\eta)C_0\}\Big] + ..., \\ R_n = R_0 + \frac{2-\xi}{2}\left\{(1-\xi) + \frac{\xi}{\xi!}t^{\xi}\right\}\{\varrho(1-\sigma)B_0 + \vartheta I_0 - (\nu+\mu)R_0\} \\ \quad + \left(\frac{(2-\xi)^2}{4}\right)\left[(1-\xi)^2 + \frac{\xi^2}{2\xi!}t^{2\xi} + \frac{2\xi(1-\xi)}{\xi!}t^{\xi}\right]\Big[\{\varrho S_0 I_0 - \sigma\varrho C_0 I_0 - (\nu+\vartheta)I_0\}\{\varrho(1-\sigma)C_0 + \vartheta\} \\ \quad + \varrho(1-\sigma)I_0\{\mu R_0 - \varrho C_0 I_0 - (\nu+\eta)C_0\} - (\nu+\mu)\{\varrho(1-\sigma)C_0 I_0 + \vartheta I_0 - (\nu+\mu)R_0\}\Big] + ..., \\ C_n = C_0 + \frac{2-\xi}{2}\left\{(1-\xi) + \frac{\xi}{\xi!}t^{\xi}\right\}\{\mu R_0 - \varrho B_0 - (\nu+\eta)C_0\} \\ \quad + \left(\frac{(2-\xi)^2}{4}\right)\left[(1-\xi)^2 + \frac{\xi^2}{2\xi!}t^{2\xi} + \frac{2\xi(1-\xi)}{\xi!}t^{\xi}\right]\Big[\mu\{\varrho(1-\sigma)C_0 I_0 + \vartheta I_0 - (\nu+\mu)R_0\} \\ \quad - \varrho C_0\{\varrho S_0 I_0 - \sigma\varrho C_0 I_0 - (\nu+\vartheta)I_0\} - \{\mu R_0 - \varrho C_0 I_0 - (\nu+\eta)C_0\}\{\varrho I_0 + (\nu+\eta)\}\Big] + ... \end{cases} \tag{5.8}$$

For the convergence of the series in (5.4), we establishe the following result

Theorem 5.1. *[57] Let $\mathbb{Z}$ be Banach space and $\mathbb{T} : \mathbb{Z} \to \mathbb{Z}$ be a contraction operator such that for all $u, \bar{u} \in \mathbb{Z}$, with $||\mathbb{T}(u) - \mathbb{T}(\bar{u})|| \leq k||u - \bar{u}||, o < k < 1$. By banach result (3.2), $\mathbb{T}$ has a unique fixed point u such that $\mathbb{T}u = u$ the series given in (5.4) may be express as*

$$U_n = TU_{n-1}, U_{n-1} = \sum_{n=0}^{j-i} U_n, \ \ where j = 1, 2, 3, ...$$

If $u_0 \in \mathbb{B}_\rho(U)$ where $\mathbb{B}_\rho(U) = \{\bar{U} \in \mathbb{Z} : ||\bar{U} - U|| < \rho\}$, then

$$(i) U_n \in \mathbb{B}_\rho(U),$$
$$(ii) lim_{n\to\infty} U_n = U$$

6. NUMERICAL SIMULATION

In this section, we provide some approximation to the parameters which are consider in the model. The following tale presents the assumed values of the parameters.
We get the following infinite series solution upto three terms for the considered system (1.1), which is based on the above values mentioned in the table, also for different values of ξ.

Parameters	Numerical values (assumed)
S_0	0.8 (Initial susceptible population)
I_0	0.1 (Initial infection population)
R_0	0.05 (Initial recovered population)
C_0	0.05 (Initial cross-immune population)
ν	0.5 (Mortality rate)
ϑ	73 (Rate of progression from infective to recovered per year)
μ	1 (Rate of progression from recovered to cross-immune per year)
η	0.5 (Rate of progression from recovered to susceptible per year)
σ	0.5 (Recruitment rate of cross-immune into the infective)
ϱ	100 (Contact rate per year)

Table 2: Parameters used in model (1.1).

By plugging $\xi = 1$ and values of parameter given in the above table (2), we obtain

$$\begin{cases} S_n = 0.8 - 3.6875t + 1.13671875t^2 + ...,, \\ I_n = 0.1 + 7.5t + 17.05625t^2 + ..., \\ R_n = 0.05 + 7.475t + 4.31875t^2 + ..., \\ C_n = 0.05 - 0.25t + 5.4875t^2 + ... \end{cases} \tag{6.1}$$

By plugging $\xi = 0.9$ and values of parameter given in the above table (2), we obtain

$$\begin{cases} S_n = 0.4218835938 - 3.280913303t^{0.9} + 1.15838805t^{1.8} + ..., \\ I_n = 1.33776125 + 15.44523834t^{0.9} + 17.38139376t^{1.8} + ..., \\ R_n = 0.20951375 + 7.972963372t^{0.9} + 4.401078448t^{1.8} + ..., \\ C_n = 0.055699375 + 1.796695144t^{0.9} + 4.159419442t^2 + ... \end{cases} \tag{6.2}$$

By plugging $\xi = 0.8$ and values of parameter given in the above table (2), we obtain

$$\begin{cases} S_n = 0.00955 - 2.676018283t^{0.8} + 1.124778027t^{1.6} + ..., \\ I_n = 3.86488 + 24.60751488t^{0.8} + 16.8770817t^{1.6} + ..., \\ R_n = 0.6752 + 85.5709324t^{0.8} + 4.273383458t^{1.6} + ..., \\ C_n = 0.14804 + 3.513052409t^{0.8} + 3.016586812t^2.... \end{cases} \tag{6.3}$$

By plugging $\xi = 0.7$ and values of parameter given in the above table (2), we obtain

$$\begin{cases} S_n = 0.2923351563 - 1.917086516t^{0.7} + 1.035963759t^{1.4} + ..., \\ I_n = 8.21351125 + 34.15884815t^{0.7} + 15.5444404t^{1.4} + ..., \\ R_n = 1.55876375 + 9.132285968t^{0.7} + 3.93595028t^{1.4} + ..., \\ C_n = 0.369824375 + 4.822598731t^{0.7} + 2.071464084t^2 + ... \end{cases} \tag{6.4}$$

Using different values of ξ and parameters given in the table (2), we obtained the following graphs using MATLAB.

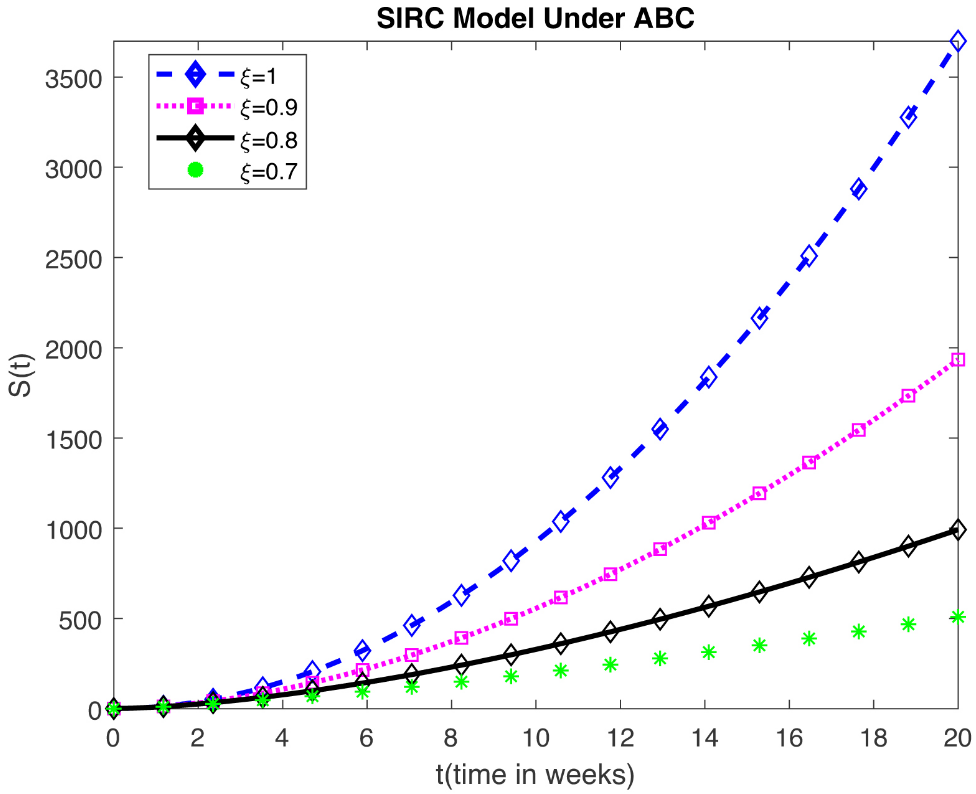

Figure 1: Plot shows the behavior of S(t) at different values of fractional order ξ.

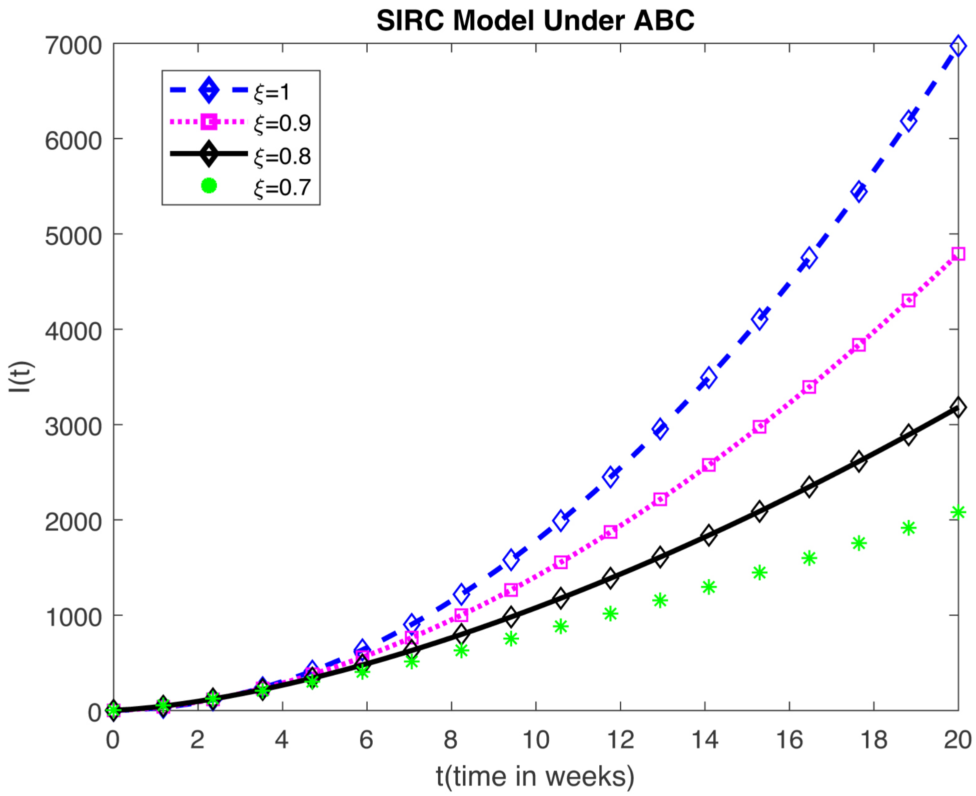

Figure 2: Plot shows the behavior of I(t) at different values of fractional order ξ.

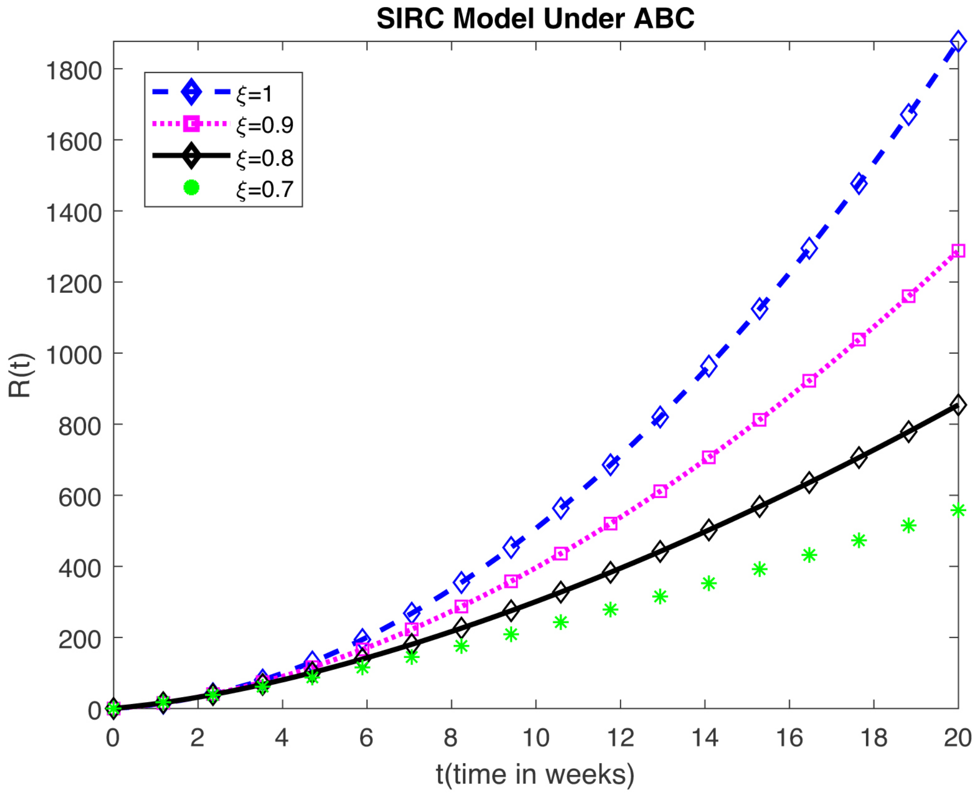

Figure 3: Plot shows the behavior of R(t) at different values of fractional order ξ.

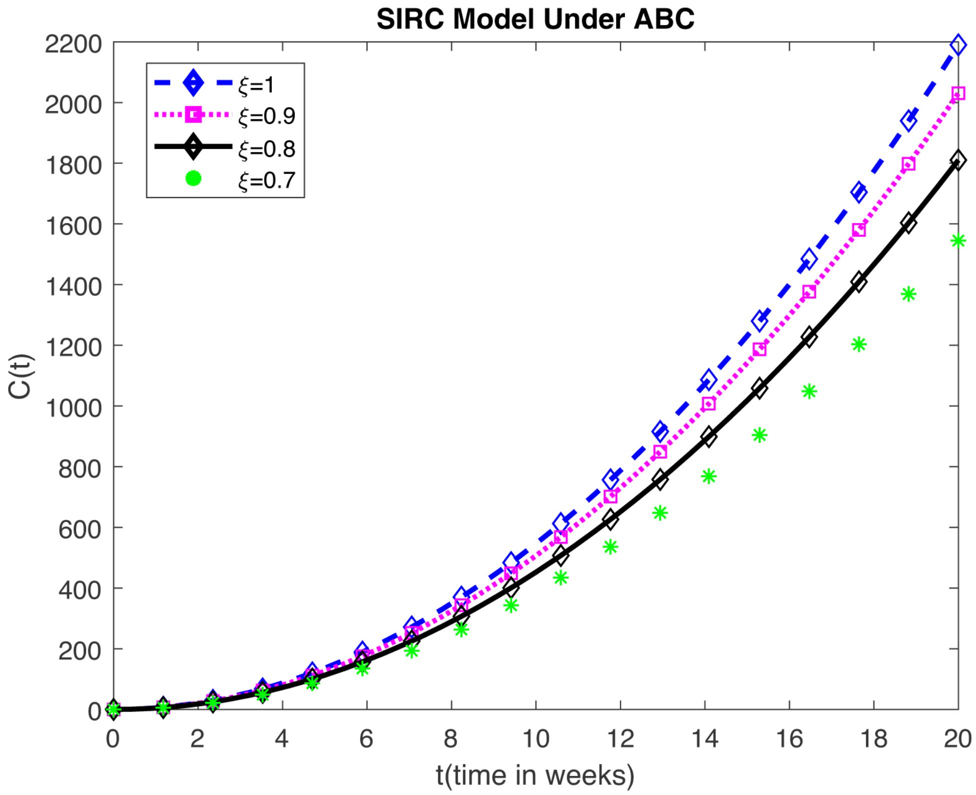

Figure 4: Plot shows the behavior of C(t) at different values of fractional order ξ.

7. Conclusion

We successfully obtained the conditions for the qualitative and approximate solution of SIRC model model under fractional order derivative with out singular kernel of *ABC* type. With the help of tools of analysis, we proved the existence results of the proposed model. The sami-analytical reults are obtained via Laplace Adomian decomposition method. To illustrate the dynamics behaviors of consider model, we also provides graphical presentations.

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