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Coincidence Points with φ-Contractions in Partially Ordered Fuzzy Metric Spaces

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Abstract

In this paper our main result is a coupled coincidence point theorem for a compatible pair of a coupled and a singled mapping in a fuzzy metric space with a partial ordering under the assumption of a new coupled contraction inequality which involves a recently introduced control function which is a generalization of many such functions previously used in literatures. The proof depends on a lemma in which we prove a condition for simultaneous holding of Cauchy criteria for two sequences. We use H-type t-norms in this paper in order to utilize the equi-continuity of the t-norm iterates in the proof of the lemma. The contraction inequality also involves another function which is borrowed from a recent work. There is a partial ordering defined on the fuzzy metric space. There are several corollaries of the main theorem. An illustrative example is given.

Keywords: Partial ordered set, *Hadžić* type t-norm, φ- function, Cauchy sequence, compatibility, coupled coincidence point.
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1. Introduction

Fuzzy fixed point theory has developed into a vast area of study during the last two and a half decades in which a large number of results have appeared in the setting of fuzzy metric spaces introduced by George and Veeramani [12] in 1994. The successful development of metric fixed point theory in this space is to a large extent due to the fact that the topology of such space is Hausdorff. Actually the authors of [12] introduced the definition of fuzzy metric space by modifying of an earlier definition which was given by Kramosil and Michalek [19] with a view to obtaining a Hausdorff topology in this space. Examples of works

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from this field of research are [7, 21]. It may be mentioned that the aforesaid development is not a mere fuzzy extension of metric fixed point theory. It has its own characteristics. Sometimes it is more versatile than the fixed point theory in metric space. This versatile character is derived from the inherent flexibility of fuzzy concepts. As an instance, we consider the case of Banach's contraction mapping principle which has several inequivalent fuzzy extensions leading to the concepts of B-contraction [26], C-contraction [15], etc.

Control functions have been used in many results in fixed point theory. Such use was initiated in metric fixed point theory by Khan, Swaleh and Sessa [18] in 1984. Essentially control function alters the metric distance which makes the triangular inequality directly unavailable in the proofs of the theorems. New methods in the proofs are then to be introduced in order to obtain the final results. The work of Khan et al [18] was following by several other works in the same vein. The idea of control function was extended by Choudhury et al [2] to fixed point theory in probabilistic metric spaces which is a sister branch of fuzzy fixed point theory. Recently Fang [10] utilized one of such functions which includes several other control functions appearing in earlier works as special cases. In our present work we use the function introduced by Fang [10] as mentioned above.

Coupled fixed point problems have attracted large attention in recent years. Although the concept was introduced by Guo et al [11] in 1987, large interest was felt in this category of problems after the coupled contraction mapping theorem was established by Bhaskar et al [1]. Several works like [3, 9, 20] followed the work of Bhaskar et al in this line of study in the metric spaces. In fuzzy metric spaces, coupled fixed point results were introduced in the work in Zhu [27] which was followed by works like [4, 5, 6, 16, 17, 23, 24].

Fixed point and related problems in partially ordered metric spaces have been studied in a good number of papers in recent times as, for instances, in [1, 3, 20]. Such problems in fuzzy metric space with a partial ordering have been considered works some of which are [4, 5, 6, 16, 17].

In this paper our main result is a coupled coincidence point theorem for a compatible pair in a fuzzy metric space with a partial ordering under the assumption of a coupled contraction inequality which involves a control function introduced by Fang [10] as mentioned earlier. The proof depends on a lemma in which we prove a condition for simultaneous holding of Cauchy criteria for two sequences. We use H-type t-norms in our results. The reason is that we utilize the equi-continuity of the t-norm iterates at '1' in the above mentioned lemma. The inequality also involves another function which is borrowed the work of Choudhury et al [4]. The main result is illustrated with an example. There are several corollaries of the main theorem. The example shows that the corollaries are properly contained in the theorem.

2. Preliminaries

Definition 2.1[14, 25] A binary operation $* : [0,1]^2 \longrightarrow [0,1]$ is said to be a t-norm if:

(i) $*$ is commutative and associative,

(ii) 1 is the unity of $*$,

(iii) $*$ is the monotone increasing in both variables,

(iv) The operator $*$ is a continuous t-norm if $*$ in continuous.

For examples of t-norm we refer to [14, 25].
The following is the definition of fuzzy metric space with which we work the definition is given by George et al [12].

Definition 2.2[12] The 3-tuple $(S, M, *)$ is said to be a fuzzy metric space if S is an arbitrary non-empty set, $*$ is a continuous t-norm and M is a fuzzy set on $S^2 \times (0, \infty)$ satisfying the following conditions for each $x_1, x_2, x_3 \in S$ and $t, s > 0$:

(i) $M(x_1, x_2, t) > 0$,

(ii) $M(x_1, x_2, t) = 1$ if and only if $x_1 = x_2$,

(iii) $M(x_1, x_2, t) = M(x_2, x_1, t)$,

(iv) $M(x_1, x_2, t) * M(x_2, x_3, s) \leq M(x_1, x_3, t + s)$ and

(v) $M(x_1, x_2, .) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Let $(S, M, *)$ be a fuzzy metric space. For $t > 0,\ \ 0 < r < 1$, the open ball $B(x_1, t, r)$ with center $x_1 \in S$ is defined by

$$B(x_1, t, r) = \{y \in S : M(x_1, x_2, t) > 1 - r\}.$$

These open balls form a base for a topology which is metrizable.

Definition 2.3[12] Let $(S, M, *)$ be a fuzzy metric space.

(i) A sequenc $\{y_n\}$ in S is convergent to $y \in S$ if $\lim_{n\to\infty} M(y_n, y, t) = 1$ for every $t > 0$.

(ii) A sequence $\{y_n\}$ in S is Cauchy sequence if for every $0 < \varepsilon < 1$ and $t > 0$, we can find a positive integer N such that $M(y_n, y_m, t) > 1 - \varepsilon$ for every $n, m \geq N$.

(iii) A fuzzy metric space is called complete when a sequence is convergent whenever it is a Cauchy sequence.

Lemma 2.4[13] Let $(S, M, *)$ be a fuzzy metric space. Then $M(x_1, x_2, .)$ is nondecreasing for all $x_1, x_2 \in S$.

Lemma 2.5[22] M is a continuous function on $S^2 \times (0, \infty)$ for a fuzzy metric space $(S, M, *)$.

Here is established a coupled coincidence point theorem for two mappings complete fuzzy metric space having a partial order defined on it.

Let $(S, \preceq)$ be a partially ordered set and F be a mapping from S to itself. The mapping F is a non-decreasing mapping if for all $\alpha_1, \alpha_2 \in S$, $\alpha_1 \preceq \alpha_2$ implies $F(\alpha_1) \preceq F(\alpha_2)$ and a non-increasing mapping if for all $\alpha_1, \alpha_2 \in S$, $\alpha_1 \preceq \alpha_2$ implies $F(\alpha_1) \succeq F(\alpha_2)$[1].

Definition 2.6[1] Let $(S, \preceq)$ be a partially ordered set and $F : S \times S \to S$ be a mapping. The mapping F is said to have the mixed monotone property if, for all $\alpha_1, \alpha_2 \in S$, $\alpha_1 \preceq \alpha_2$ implies $F(\alpha_1, \beta) \preceq F(\alpha_2, \beta)$, for fixed $\beta \in S$ and, for all $\beta_1, \beta_2 \in S$, $\beta_1 \preceq \beta_2$ implies $F(\alpha, \beta_1) \succeq F(\alpha, \beta_2)$, for fixed $\alpha \in S$.

Definition 2.7[20] Let $(S, \preceq)$ be a partially ordered set and $F : S \times S \to S$ and $g : S \to S$ be two mappings. The mapping F has the mixed g-monotone property if, for all $\alpha_1, \alpha_2 \in S$, $g(\alpha_1) \preceq g(\alpha_2)$ implies $F(\alpha_1, \beta) \preceq F(\alpha_2, \beta)$, for any $\beta \in S$ and, for all $\beta_1, \beta_2 \in S$, $g(\beta_1) \preceq g(\beta_2)$ implies $F(\alpha, \beta_1) \succeq F(\alpha, \beta_2)$, for any $\alpha \in S$.

Definition 2.8[20] Let S be a nonempty set. An element $(\alpha, \beta) \in S \times S$ is called a coupled coincidence point of the mappings $F : S \times S \to S$ and $g : S \to S$ if

$$F(\alpha, \beta) = g(\alpha) \text{ and } F(\beta, \alpha) = g(\beta).$$

If $g = I$, the identity mapping, then we have a coupled fixed point (α, β), that is, $F(\alpha, \beta) = \alpha$ and $F(\beta, \alpha) = \beta$.

Definition 2.9[20] Let S be a nonempty set. The mappings $F : S \times S \to S$ and $g : S \to S$ are commuting if for every $\alpha, \beta \in S$,

$$g(F(\alpha, \beta)) = F(g(\alpha), g(\beta)).$$

Definition 2.10[16] Let $(S, M, *)$ be a fuzzy metric space. The mappings F and g where $F : S \times S \to S$ and $g : S \to S$, are called to be compatible if for every $t > 0$

$$\lim_{n\to\infty} M(g(F(\alpha_n, \beta_n)), F(g(\alpha_n), g(\beta_n), t) = 1$$

and

$$\lim_{n\to\infty} M(g(F(\beta_n, \alpha_n)), F(g(\beta_n), g(\alpha_n), t) = 1,$$

whenever $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in S such that $\lim_{n\to\infty} F(\alpha_n, \beta_n) = \lim_{n\to\infty} g(\alpha_n) = \alpha$ and $\lim_{n\to\infty} F(\beta_n, \alpha_n) = \lim_{n\to\infty} g(\beta_n) = \beta$ for some $\alpha, \beta \in S$.

Note. A commuting mapping is compatible, but not the converse.

Definition 2.11 [8] Let Φ_w represent the set of all mappings $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying the following condition:

$$\lim_{n\to\infty} \varphi^n(t) = 0 \text{ for every } t > 0.$$

Definition 2.12 [10] Let Φ represent the set of all mappings $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying the following condition:

$$\text{for each } t > 0 \text{ there exists } r \geq t \text{ such that } \lim_{n\to\infty} \varphi^n(r) = 0.$$

Here Φ_w is a proper subclass of Φ [see [10]].

Lemma 2.13 [10] Let $\varphi \in \Phi$, then for each $t > 0$ there exists $r \geq t$ such that $\varphi(r) < t$.

Definition 2.14[14] A t-norm $*$ is a *Hadžić* type t-norm if the family $\{*^p\}_{p\geq 0}$ of its iterates defined for each $s \in [0, 1]$ by

$*^0(s) = 1$, $*^{p+1}(s) = *(*^p(s), s)$ for all $p \geq 0$, is equi-continuous at $s = 1$, that is, given $\lambda > 0$ there exists $\eta(\lambda) \in (0, 1)$ such that

$1 \geq s > \eta(\lambda) \Rightarrow *^{(p)}(s) > 1 - \lambda$ for all $p \geq 0$.

Examples of the above definition are obtainable in [14].

3. Main Results

Lemma 3.1 Let $(S, M, *)$ be a fuzzy metric space with a *Hadžić* type t-norm $*$ for which $M(a, b, t) \to 1$ as $t \to \infty$, for all $a, b \in S$. If $\{a_n\}$ and $\{b_n\}$ in S are such that, for all $n \geq 1$, $t > 0$,

$$M(a_n, a_{n+1}, \varphi(t)) * M(b_n, b_{n+1}, \varphi(t)) \geq M(a_{n-1}, a_n, t) * M(b_{n-1}, b_n, t) \tag{3.1}$$

where $\varphi \in \Phi$, then $\{a_n\}$ and $\{b_n\}$ are Cauchy sequences.

Proof. By repeated applications of (3.1) it follows that for every $t > 0$, $n \geq 1$,

$$M(a_n, a_{n+1}, \varphi^n(t)) * M(b_n, b_{n+1}, \varphi^n(t)) \geq M(a_0, a_1, t) * M(b_0, b_1, t). \tag{3.2}$$

Now we prove that $\lim_{n\to\infty} M(a_n, a_{n+1}, t) * M(b_n, b_{n+1}, t) = 1$, for every $t > 0$. (3.3)

Since $M(a_0, a_1, t) \to 1$ and $M(b_0, b_1, t) \to 1$ as $t \to \infty$, and $*$ is continuous, we have that given $\epsilon \in (0, 1]$ there exists $t_1 > 0$ such that

$$M(a_0, a_1, t_1) * M(b_0, b_1, t_1) > 1 - \epsilon. \tag{3.4}$$

Since $\varphi \in \Phi$, there exists $t_0 \geq t_1$ such that $\lim_{n\to\infty} \varphi^n(t_0) = 0$. then, for every $t > 0$, there exists an integer $n_0 \geq 1$ such that $\varphi^n(t_0) < t$ for every $n \geq n_0$.

Then, we get for every $n \geq n_0$,

$$\begin{aligned} M(a_n, a_{n+1}, t) * M(b_n, b_{n+1}, t) &\geq M(a_n, a_{n+1}, \varphi^n(t_0)) * M(b_n, b_{n+1}, \varphi^n(t_0)) \\ &\geq M(a_0, a_1, t_0) * M(b_0, b_1, t_0) \quad \text{(by (3.2))} \\ &\geq M(a_0, a_1, t_1) * M(b_0, b_1, t_1) \\ &> 1 - \epsilon. \quad \text{(by (3.4))} \end{aligned}$$

Therefore (3.3) holds.

$$\text{Let } P_n(t) = M(a_n, a_{n+1}, t) * M(b_n, b_{n+1}, t). \tag{3.5}$$

Since $\varphi \in \Phi$, by Lemma 2.13, for any $t > 0$ there exists $r \geq t$

such that $\varphi(r) < t$.
Let $n \geq 1$ be given. We next show by induction that for any $k \geq 1$,

$$M(a_n, a_{n+k}, t) * M(b_n, b_{n+k}, t) \geq *^{k-1}(P_n(t - \varphi(r))). \tag{3.6}$$

Since $*^0(s) = s$, this true for $k = 1$. Let (3.6) holds for some k.
For any $t > 0$, there exists a positive integer n_0 such that for every $n \geq n_0$,

$$\begin{aligned} &M(a_n, a_{n+k+1}, t) * M(b_n, b_{n+k+1}, t) \\ &= M(a_n, a_{n+k+1}, (t - \varphi(r) + \varphi(r))) * M(b_n, b_{n+k+1}, (t - \varphi(r) + \varphi(r))) \\ &\geq \{M(a_n, a_{n+1}, (t - \varphi(r))) * M(b_n, b_{n+1}, (t - \varphi(r)))\} \\ &\quad *\{M(a_{n+1}, a_{n+k+1}, \varphi(r)) * M(b_{n+1}, b_{n+k+1}, \varphi(r))\} \\ &\geq P_n(t - \varphi(r)) * \{M(a_n, a_{n+k}, r) * M(b_n, b_{n+k}, r)\} \quad \text{(by (3.1) and (3.5))} \\ &\geq P_n(t - \varphi(r)) * \{M(a_n, a_{n+k}, t) * M(b_n, b_{n+k}, t)\} \quad \text{(since } r \geq t) \\ &\geq P_n(t - \varphi(r)) * \{*^{k-1}(P_n(t - \varphi(r)))\} \quad \text{(since by our assumption (3.6) holds for integer k)} \\ &\geq *^k P_n(t - \varphi(r)). \end{aligned}$$

Therefore, by induction, for all $n \geq 1$, (3.6) holds for all $k \geq 1$ and $t > 0$.
Since, the t-norm $*$ is of H-type, so for given $\epsilon > 0$, there exists $\delta \in (0, 1)$ such that

$$*^p(s) > 1 - \epsilon, \tag{3.7}$$

whenever $1 \geq s > 1 - \delta$ and $p \geq 1$. (3.8)
By (3.3) and (3.5), we have

$$\lim_{n\to\infty} P_n(t - \varphi(r)) = \lim_{n\to\infty} M(a_n, a_{n+1}, (t - \varphi(r))) * M(b_n, b_{n+1}, (t - \varphi(r))) = 1.$$

Then, there exists a positive integer n_0 such that for all $n \geq n_0$, we have
$P_n(t - \varphi(r) = M(a_n, a_{n+1}, (t - \varphi(r))) * M(b_n, b_{n+1}, (t - \varphi(r))) > 1 - \delta$ when δ is the same as in (3.8). (3.9)
Taking $s = P_n(t - \varphi(r)$, it follows from (3.6)-(3.9) that
$M(a_n, a_{n+k}, t) * M(b_n, b_{n+k}, t) \geq *^{k-1}(P_n(t - \varphi(r))) > 1 - \epsilon$ for all $n \geq n_0$ and $k \geq 1$.
The above inequality implies that

$$M(a_n, a_m, t) > 1 - \epsilon \text{ and } M(b_n, b_m, t) > 1 - \epsilon \text{ for all } n, m > n_0.$$

This shows that $\{a_n\}$ and $\{b_n\}$ are Cauchy sequences.

Next we prove that the main theorem of the paper.

Theorem 3.2 Let $(S, M, *)$ be a complete fuzzy metric space, where $*$is *Hadžić* type t-norm such that $M(a, b, t) \to 1$ as $t \to \infty$, for every $a, b \in S$. Let $\preceq$ be a partial order defined on S. Let $F : S \times S \to S$ and $g : S \to S$ be two mappings such that F has mixed g-monotone property and satisfies the following condition:

$$M(F(a, b), F(u, v), \varphi(t)) \geq \gamma(M(g(a), g(u), t) * M(g(b), g(v), t)), \tag{3.10}$$

for all $a, b, u, v \in S$, $t > 0$ with $g(a) \preceq g(u)$ and $g(b) \succeq g(v)$, where $\varphi \in \Phi$, $\gamma : [0, 1] \to [0, 1]$ is a continuous function such that $\gamma(x) * \gamma(x) \geq x$ for each $0 \leq x \leq 1$ and $F(S \times S) \subseteq g(S)$, g is continuous and monotonic increasing, (g, F) is a compatible pair. Also suppose that S has the following properties:

(i) if a sequence $\{a_n\} \to x$ is such that $a_n \preceq a_{n+1}$ for every $n \geq 0$, then $a_n \preceq a$ for all $n \geq 0$, (3.11)

(ii) if a sequence $\{b_n\} \to b$ is such that $b_n \succeq b_{n+1}$ for all $n \geq 0$, then $b_n \succeq b$ for every $n \geq 0$. (3.12)

If there are $a_0, b_0 \in S$ such that $g(a_0) \preceq F(a_0, b_0)$, $g(b_0) \succeq F(b_0, a_0)$, then there exist $a, b \in S$ such that $g(a) = F(a, b)$ and $g(b) = F(b, a)$.

Proof. Starting with a_0, b_0 in S and since $F(S \times S) \subseteq g(S)$, we define $\{a_n\}$ and $\{b_n\}$ in S as follows:

$$g(a_1) = F(a_0, b_0) \text{ and } g(b_1) = F(b_0, a_0),$$
$$g(a_2) = F(a_1, b_1) \text{ and } g(b_2) = F(b_1, a_1),$$

and in general, for every $n \geq 0$,

$$g(a_{n+1}) = F(a_n, b_n) \text{ and } g(b_{n+1}) = F(b_n, a_n). \tag{3.13}$$

Next, we prove that $\{g(a_n)\}$ and $\{g(b_n)\}$ are monotone increasing and decreasing sequences respectively, that is, for every $n \geq 0$,

$$g(a_n) \preceq g(a_{n+1}) \tag{3.14}$$

and

$$g(b_n) \succeq g(b_{n+1}). \tag{3.15}$$

From the conditions on a_0, b_0, we have $g(a_0) \preceq F(a_0, b_0) = g(a_1)$
and $g(b_0) \succeq F(b_0, a_0) = g(b_1)$. Therefore (3.14) and (3.15) hold every $n = 0$.
Let (3.14) and (3.15) hold for some $n = m$. As F has the mixed g-monotone property and $g(a_m) \preceq g(a_{m+1})$, $g(b_m) \succeq g(b_{m+1})$, it follows that
$g(a_{m+1}) = F(a_m, b_m) \preceq F(a_{m+1}, b_m)$ and $F(b_{m+1}, a_m) \preceq F(b_m, a_m) = g(b_{m+1})$.
Also, for the same reason, we have
$F(a_{m+1}, b_m) \preceq F(a_{m+1}, b_{m+1}) = g(a_{m+2})$ and $g(b_{m+2}) = F(b_{m+1}, a_{m+1}) \preceq F(b_{m+1}, a_m)$.
Then, from the above two relations it follows that

$$g(a_{m+1}) \preceq g(a_{m+2}) \text{ and } g(b_{m+1}) \succeq g(b_{m+2}).$$

Then, by induction, (3.14) and (3.15) hold for every $n \geq 0$.
Let for every $t > 0$, $n \geq 0$,

$$\delta_n(t) = M(g(a_n), g(a_{n+1}), t) * M(g(b_n), g(b_{n+1}), t). \tag{3.16}$$

Then, for every $t > 0$, $n \geq 1$, we have

$$\begin{aligned} M(g(a_n), g(a_{n+1}), \varphi(t)) &= M(F(a_{n-1}, b_{n-1}), F(a_n, b_n), \varphi(t))(\text{by } (3.13)) \\ &\geq \gamma(M(g(a_{n-1}), g(a_n), t) * M(g(b_{n-1}), g(b_n), t))(\text{by } (3.10)) \\ &= \gamma(\delta_{n-1}(t)).(\text{by } (3.16)) \end{aligned} \tag{3.17}$$

Similarity, for every $t > 0$, $n \geq 1$, we get

$$\begin{aligned} M(g(b_n), g(b_{n+1}), \varphi(t)) &= M(F(b_{n-1}, a_{n-1}), F(b_n, a_n), \varphi(t))(\text{by } (3.13)) \\ &\geq \gamma(M(g(b_{n-1}), g(b_n), t) * M(g(a_{n-1}), g(a_n), t)) \ (\text{by } (3.10)) \\ &= \gamma(\delta_{n-1}(t)). \ (\text{by } (3.16)) \end{aligned} \tag{3.18}$$

From (3.14) and (3.15), for every $t > 0$, $n \geq 1$, it follows that

$$\begin{aligned} M(g(a_n), g(a_{n+1}), \varphi(t)) * M(g(b_n), g(b_{n+1}), \varphi(t)) &\geq \gamma(\delta_{n-1}(t)) * \gamma(\delta_{n-1}(t)) \\ &\geq \delta_{n-1}(t) \ (\text{since } \gamma(x) * \gamma(x) \geq x), \end{aligned}$$

that is, $M(g(a_n), g(a_{n+1}), \varphi(t)) * M(g(b_n), g(b_{n+1}), \varphi(t))$

$$\geq M(g(a_{n-1}), g(a_n), t) * M(g(b_{n-1}), g(b_n), t). \ (\text{by}(3.16)) \tag{3.19}$$

Then from Lemma 3.1, using (3.19) that $\{g(a_n)\}$ and $\{g(b_n)\}$ are Cauchy sequences. Since S is complete, there exist $a, b \in S$ such that

$$\lim_{n\to\infty} g(a_n) = a \text{ and } \lim_{n\to\infty} g(b_n) = b, \tag{3.20}$$

that is, $\lim_{n\to\infty} g(a_{n+1}) = \lim_{n\to\infty} F(a_n, b_n) = a$ and $\lim_{n\to\infty} g(b_{n+1}) = \lim_{n\to\infty} F(b_n, a_n) = b.$ (3.21)

Then, from the compatibility of g and F (Definition 2.10), we have

$$g(a) = \lim_{n\to\infty} g(g(a_{n+1})) = \lim_{n\to\infty} g(F(a_n, b_n)) = \lim_{n\to\infty} F(g(a_n), g(b_n)) \tag{3.22}$$

and

$$g(b) = \lim_{n\to\infty} g(g(b_{n+1})) = \lim_{n\to\infty} g(F(b_n, a_n)) = \lim_{n\to\infty} F(g(b_n), g(a_n)). \tag{3.23}$$

Also from (3.14), (3.15) and (3.20), and by (3.11) and (3.12), we get for every $n \geq 0$,

$$g(a_n) \preceq a \text{ and } g(b_n) \succeq b.$$

Since g is monotonic increasing,

$$g(g(a_n)) \preceq g(a) \text{ and } g(g(b_n)) \succeq g(b). \tag{3.24}$$

For each $t > 0$, by Lemma 2.13, there exists $r \geq t$ such that $\varphi(r) < t$. Now, for each $t > 0$, $n \geq 0$, and $r \geq t$ as in the above, we have

$$M(g(a), F(a, b), t) \quad \geq \quad M(g(a), g(g(a_{n+1})), (t - \varphi(r))) * M(g(g(a_{n+1})), F(a, b), \varphi(r)).$$

Taking $n \to \infty$ on the both sides of the above inequality, for all $t > 0$,

$$\begin{aligned} M(F(a, b), g(a), t) &\geq \lim_{n\to\infty} [M(g(a), g(g(a_{n+1})), (t - \varphi(r))) * M(g(g(a_{n+1})), F(a, b), \varphi(r))] \\ &= M(g(a), \lim_{n\to\infty} g(g(a_{n+1})), (t - \varphi(r))) \\ &* M(\lim_{n\to\infty} g(F(a_n, b_n)), F(a, b), \varphi(r)) \\ &\qquad (\text{by Lemma } 2.5) \\ &= M(g(a), g(a), (t - \varphi(r))) * M(\lim_{n\to\infty} (F(g(a_n), g(b_n))), F(a, b), \varphi(r)) \end{aligned}$$

$$\begin{aligned}&\text{(by (3.22) and the compatibility of } g \text{ and } F)\\ &= 1 * \lim_{n\to\infty} M(F(g(a_n), g(b_n)), F(a,b), \varphi(r)) \quad \text{(by Lemma 2.5)}\\ &= \lim_{n\to\infty} M(F(g(a_n), g(b_n)), F(a,b), \varphi(r)).\end{aligned}$$

Again, by virtue of (3.24), using the inequality (3.10), we have
$M(F(g(a_n), g(b_n))), F(a,b), \varphi(r)) \geq \gamma(M(g(g(a_n)), g(a), r) * M(g(g(b_n)), g(b), r))$.
Then, from the above two inequalities, for all $t > 0$, we obtain

$$\begin{aligned} M(F(a,b), g(a), t) &\geq \lim_{n\to\infty}[\gamma(M(g(g(a_n)), g(a), r) * M(g(g(b_n)), g(b), r))]\\ &= \gamma(M(\lim_{n\to\infty} g(g(a_n)), g(a), r) * M(\lim_{n\to\infty} g(g(b_n)), g(b), r))\\ &\qquad \text{(Since } \gamma \text{ and M are both continuous)}\\ &= \gamma(M(g(a), g(a), r) * M(g(b), g(b), r)) \quad (by (3.22)\\ &= \gamma(1*1)\\ &= \gamma(1)\\ &= 1,\end{aligned}$$

this shows that $g(a) = F(a,b)$.
By similar application of (3.23), we get $g(b) = F(b,a)$.

Remark 3.3 In the above theorem we use a function γ which has a property involving the t-norm $*$. It is a question whether such functions exist for a given t-norm. This function has been used in [4] wherein it has been shown that such function exists with respect to arbitrary continuous *Hadžić* type t-norm.

Corollary 3.4 Let $(S, M, *)$ be a complete fuzzy metric space with a *Hadžić* type t-norm, such that $M(a,b,t) \to 1$ as $t \to \infty$, for all $a, b \in S$ and with a partially order $\preceq$ defined on it. Let $F : S \times S \to S$ and $g : S \to S$ be two mappings such that F has mixed g-monotone property and satisfies (3.10), for every $a, b, u, v \in S,\ t > 0$ with $g(a) \preceq g(u)$ and $g(b) \succeq g(v)$, where $\varphi \in \Phi_w$, $\gamma : [0,1] \to [0,1]$ is a continuous function such that $\gamma(x) * \gamma(x) \geq x$ for each $0 \leq x \leq 1$ and $F(S \times S) \subseteq g(S)$, g is continuous and monotonic increasing, (g, F) is a compatible pair. Also suppose that S satisfies (3.11) and (3.12).
If there are $a_0, b_0 \in S$ such that $g(a_0) \preceq F(a_0, b_0)$, $g(b_0) \succeq F(b_0, a_0)$, then there exist $a, b \in S$ such that $g(a) = F(a,b)$ and $g(b) = F(b,a)$.
Proof. Since Φ_w is a proper subclass of Φ, so the Theorem 3.2 implies the Corollary 3.4.

Corollary 3.5 Let $(S, M, *)$ be a complete fuzzy metric space with a *Hadžić* type t-norm such that $M(a,b,t) \to 1$ as $t \to \infty$, for all $a, b \in S$. Let $\preceq$ be a partial order defined on S. Let $F : S \times S \to S$ and $g : S \to S$ be two mappings such that F has mixed g-monotone property and satisfies (3.10), for every $a, b, u, v \in S,\ t > 0$ with $g(a) \preceq g(u)$ and $g(b) \succeq g(v)$, where $\varphi \in \Phi$, $\gamma : [0,1] \to [0,1]$ is a continuous function such that $\gamma(x) * \gamma(x) \geq x$ for each $0 \leq x \leq 1$ and $F(S \times S) \subseteq g(S)$, g is continuous and monotonic increasing, (g, F) is a commuting pair. Also suppose that S satisfies (3.11) and (3.12).
If there are $a_0, b_0 \in S$ such that $g(a_0) \preceq F(a_0, b_0)$, $g(b_0) \succeq F(b_0, a_0)$, then there exist $a, b \in S$ such that $g(a) = F(a,b)$ and $g(b) = F(b,a)$.
Proof. Since a commuting pair is also a compatible pair, the result of the Corollary 3.5 follows from Theorem 3.2.

Corollary 3.4 Let $(S, \preceq)$ be a partially ordered set and let $(S, M, *)$ be a complete fuzzy metric space with a *Hadžić* type t-norm such that $M(a,b,t) \to 1$ as $t \to \infty$, for all $a, b \in S$. Let $\preceq$ be a partial order defined on S. Let $F : S \times S \to S$ be a mapping such that F has mixed monotone property and satisfies the following condition:

$$M(F(a,b), F(u,v), \varphi(t)) \geq \gamma(M(a,u,t) * M(b,v,t))$$

for all $a, b, u, v \in S,\ t > 0$ with $a \preceq u$ and $b \succeq v$, where $\varphi \in \Phi$, $\gamma : [0,1] \to [0,1]$ is a continuous function

such that $\gamma(x) * \gamma(x) \geq x$ for each $0 \leq x \leq 1$. Also suppose that S satisfies (3.11) and (3.12).
If there exist $a_0, b_0 \in S$ such that $a_0 \preceq F(a_0, b_0)$, $b_0 \succeq F(b_0, a_0)$, then there exist $a, b \in S$ such that $a = F(a, b)$ and $b = F(b, a)$.
Proof. The proof follows by putting $g = I$, the identity function, in Theorem 3.2.

4. An illustration

In this section we give an illustration of our main result. With the help of this illustration we show that the corollaries are properly included in the main theorem.

Example 4.1 Let $(S, \preceq)$ is the partially ordered set where $S = [0, 1]$ and $\preceq$ is the natural ordering $\leq$ of the real numbers. Let, for all $t > 0$, $a, b \in S$,

$$M(a, b, t) = \psi(t)^{|a-b|}$$

where ψ is an increasing and continuous function of R^+ into $(0, 1)$ given by $\psi(t) = \frac{t}{t+1}$ for each $t \in (0, \infty)$. Let $p * q = \min\{p, q\}$ for all $p, q \in [0, 1]$. Then $(S, M, *)$ is a complete fuzzy metric space such that $M(a, b, t) \to 1$ as $t \to \infty$, for all $a, b \in S$. We define the mapping $F : S \times S \to S$ as:

$$\begin{aligned} F(a, b) &= \frac{a^2 - b^2}{24}, \text{ for all } a \geq b \\ &= 0, \qquad \text{otherwise.} \end{aligned}$$

Let $g : S \to S$ be given by $g(a) = a^2$ for all $a \in S$.
Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be defined by

$$\varphi(t) = \begin{cases} \frac{t}{t+1}, & \text{if } t \in [0, 1), \\ -\frac{t}{3} + \frac{4}{3}, & \text{if } t \in [1, 2], \\ t + \frac{4}{3}, & \text{otherwise .} \end{cases}$$

It is obvious $\varphi \in \Phi$ but $\varphi \notin \Phi_w$. From the definition of φ, we have $\varphi(t) \geq \frac{t}{t+1}$ for all $t \geq 0$.
Then, clearly, $F(S \times S) \subseteq g(S)$ and F has mixed g-monotone property.
Let $\gamma : [0, 1] \to [0, 1]$ be defined as $\gamma(s) = s$ for each $s \in [0, 1]$. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be two sequences in S such that

$$\begin{aligned} &\lim_{n\to\infty} F(\alpha_n, \beta_n) = a, \quad \lim_{n\to\infty} g(\alpha_n) = a, \\ &\lim_{n\to\infty} F(\beta_n, \alpha_n) = b \text{ and } \lim_{n\to\infty} g(\beta_n) = b. \end{aligned}$$

Now, for every $n \geq 0$,

$$\begin{aligned} g(\alpha_n) &= \alpha_n^2, \;\; g(\beta_n) = \beta_n^2, \\ F(\alpha_n, \beta_n) &= \tfrac{\alpha_n^2 - \beta_n^2}{24}, \quad \text{if} \quad \alpha_n > \beta_n, \\ &= 0, \quad \text{otherwise,} \end{aligned}$$

and

$$\begin{aligned} F(\beta_n, \alpha_n) &= 0, \quad \text{if} \quad \alpha_n > \beta_n, \\ &= \tfrac{\beta_n^2 - \alpha_n^2}{24}, \quad \text{otherwise.} \end{aligned}$$

Then necessarily $a = 0$ and $b = 0$.
For all $t > 0$,

$$\lim_{n\to\infty} M(g(F(\alpha_n, \beta_n)), F(g(\alpha_n), g(\beta_n)), t) = 1$$

and

$$\lim_{n\to\infty} M(g(F(\beta_n, \alpha_n)), F(g(\beta_n), g(\alpha_n)), t) = 1.$$

Therefore the pair (g, F) is compatible in S.
Let $a_0 = 0$ and $b_0 = d > 0$.
Then $g(a_0) = g(0) = 0 = F(0, d) = F(a_0, b_0)$ and $g(b_0) = g(d) = d^2 > \frac{d^2}{24} = F(d, 0) = F(b_0, a_0)$.
Thus a_0 and b_0 satisfy their requirements in Theorem 3.2. Let $a, b, u, v \in S$ be such that $g(a) \leq g(u)$ and $g(b) \geq g(v)$, that is, $a \leq u$ and $b \geq v$.
We next show that (3.10) holds under the above condition.
The following cases may arise:

Case I. When $t \in [0,1)$, we have

$$\begin{aligned} & t(t-1)-1 \leq 0, \text{ that is, } t^2-t-1 \leq 0, \\ & \quad \text{that is, } 2t^2+t \leq 1+2t+t^2, \\ & \quad \text{that is, } t(2t+1) \leq (1+t)^2, \\ & \quad \text{that is, } \tfrac{1}{2t+1} \geq \tfrac{t}{(1+t)^2}, \\ & \quad \text{that is, } \tfrac{t}{2t+1} \geq \tfrac{t^2}{(1+t)^2} \geq \tfrac{t^4}{(1+t)^4}. \end{aligned} \tag{4.1}$$

Subcase I. $a \geq b$ and $u \geq v$.
we have $F(a,b) = \frac{a^2-b^2}{24}$, $F(u,v) = \frac{u^2-v^2}{24}$.

$$\begin{aligned} M(F(a,b),F(u,v),\varphi(t)) &= (\frac{\varphi(t)}{\varphi(t)+1})^{|F(a,b)-F(u,v)|} \\ &\geq (\frac{\frac{t}{t+1}}{\frac{t}{t+1}+1})^{|F(a,b)-F(u,v)|} \\ &= (\frac{t}{2t+1})^{|F(a,b)-F(u,v)|} \\ &= (\frac{t}{2t+1})^{\frac{|(a^2-u^2)-(b^2-v^2)|}{24}} \\ &\geq (\frac{t}{2t+1})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{8}} \\ &\geq (\frac{t}{t+1})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{2}} \quad \text{(by using (4.1))} \\ &\geq \sqrt{(\frac{t}{t+1})^{|a^2-u^2|}.(\frac{t}{t+1})^{|b^2-v^2|}} \\ &= \min\{(\frac{t}{t+1})^{|a^2-u^2|},(\frac{t}{t+1})^{|b^2-v^2|}\} \\ &= M(a^2,u^2,t)*M(b^2,v^2,t) \\ &= M(g(a),g(u),t)*M(g(b),g(v),t). \end{aligned}$$

Subcase II. $a \leq b$ and $u \geq v$.
we have $F(a,b) = 0$, $F(u,v) = \frac{u^2-v^2}{24}$.
We have $a \leq u$, it then follows that $|u^2-v^2| \leq |a^2-u^2|$,
that is, $|u^2-v^2| \leq |a^2-u^2| + |b^2-v^2|$,
that is, $\frac{|u^2-v^2|}{24} \leq \frac{1}{24}[|a^2-u^2|+|b^2-v^2|]$. (4.2)

$$\begin{aligned} M(F(a,b),F(u,v),\varphi(t)) &= (\frac{\varphi(t)}{\varphi(t)+1})^{|F(a,b)-F(u,v)|} \\ &\geq (\frac{\frac{t}{t+1}}{\frac{t}{t+1}+1})^{|F(a,b)-F(u,v)|} \\ &= (\frac{t}{2t+1})^{|F(u,v)|} \quad \text{(since F(x,y) = 0)} \\ &= (\frac{t}{2t+1})^{\frac{|u^2-v^2|}{24}} \\ &\geq (\frac{t}{2t+1})^{\frac{|(a^2-u^2)-(b^2-v^2)|}{24}} \quad \text{(by using (4.2))} \\ &\geq (\frac{t}{2t+1})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{8}} \\ &\geq (\frac{t}{t+1})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{2}} \quad \text{(by using (4.1))} \\ &\geq \sqrt{(\frac{t}{t+1})^{|a^2-u^2|}.(\frac{t}{t+1})^{|b^2-v^2|}} \\ &= \min\{(\frac{t}{t+1})^{|a^2-u^2|},(\frac{t}{t+1})^{|b^2-v^2|}\} \\ &= M(a^2,u^2,t)*M(b^2,v^2,t) \\ &= M(g(a),g(u),t)*M(g(b),g(v),t). \end{aligned}$$

Subcase III. $a \leq b$ and $u \leq v$.
$F(a,b) = 0,\ F(u,v) = 0$. Then obvious.

Case II. When $t \in [1,2]$, we have

$$-6t + 9t + 11t + 4 \geq 0,$$

that is, $-t^4 + 7t^3 - 6t + 9t + 11t + 4 \geq -t^4 + 7t^3$,
that is, $(-t+4)(1+3t+3t^2+t^3) \geq t^3(-t+7)$
that is, $\frac{-t+4}{-t+7} \geq \frac{t^3}{(1+3t+3t^2+t^3)}$
that is, $\frac{-t+4}{-t+7} \geq \frac{t^3}{(1+t)^3} \geq \frac{t^4}{(1+t)^4}$. (4.3)

Subcase I. $a \geq b$ and $u \geq v$.
we have $F(a,b) = \frac{a^2-b^2}{24}$, $F(u,v) = \frac{u^2-v^2}{24}$.

$$\begin{aligned}
M(F(a,b),F(u,v),\varphi(t)) &= (\frac{\varphi(t)}{\varphi(t)+1})^{|F(a,b)-F(u,v)|} \\
&\geq (\frac{-\frac{t}{3}+\frac{4}{3}}{-\frac{t}{3}+\frac{4}{3}+1})^{|F(a,b)-F(u,v)|} \\
&= (\frac{-t+4}{-t+7})^{|F(a,b)-F(u,v)|} \\
&= (\frac{-t+4}{-t+7})^{\frac{|(a^2-u^2)-(b^2-v^2)|}{24}} \\
&\geq (\frac{-t+4}{-t+7})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{8}} \\
&\geq (\frac{t}{t+1})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{2}} \quad \text{(by using (4.3))} \\
&\geq \sqrt{(\frac{t}{t+1})^{|a^2-u^2|}.(\frac{t}{t+1})^{|b^2-v^2|}} \\
&= \min\{(\frac{t}{t+1})^{|a^2-u^2|}, (\frac{t}{t+1})^{|b^2-v^2|}\} \\
&= M(a^2,u^2,t) * M(b^2,v^2,t) \\
&= M(g(a),g(u),t) * M(g(b),g(v),t).
\end{aligned}$$

Subcase II. $a \leq b$ and $u \geq v$.
we have $F(a,b) = 0,\ F(u,v) = \frac{u^2-v^2}{24}$.
We have $a \leq u$, it then follows that $|u^2 - v^2| \leq |a^2 - u^2|$,
that is, $|u^2 - v^2| \leq |a^2 - u^2| + |b^2 - v^2|$,
that is, $\frac{|u^2-v^2|}{24} \leq \frac{1}{24}[|a^2 - u^2| + |b^2 - v^2|]$. (4.4)

$$\begin{aligned}
M(F(a,b),F(u,v),\varphi(t)) &= (\frac{\varphi(t)}{\varphi(t)+1})^{|F(a,b)-F(u,v)|} \\
&\geq (\frac{-\frac{t}{3}+\frac{4}{3}}{-\frac{t}{3}+\frac{4}{3}+1})^{|F(a,b)-F(u,v)|} \\
&= (\frac{-t+4}{-t+7})^{|F(u,v)|} \quad \text{(since F(a,b) = 0)} \\
&= (\frac{-t+4}{-t+7})^{\frac{|u^2-v^2|}{24}} \\
&\geq (\frac{-t+4}{-t+7})^{\frac{|(a^2-u^2)-(b^2-v^2)|}{24}} \quad \text{(by using (4.4))} \\
&\geq (\frac{-t+4}{-t+7})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{8}} \\
&\geq (\frac{t}{t+1})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{2}} \quad \text{(by using (4.3))} \\
&\geq \sqrt{(\frac{t}{t+1})^{|a^2-u^2|}.(\frac{t}{t+1})^{|b^2-v^2|}}
\end{aligned}$$

$$
\begin{aligned}
&= \min\{(\frac{t}{t+1})^{|a^2-u^2|}, (\frac{t}{t+1})^{|b^2-v^2|}\} \\
&= M(a^2, u^2, t) * M(b^2, v^2, t) \\
&= M(g(a), g(u), t) * M(g(b), g(v), t).
\end{aligned}
$$

Subcase III. $a \leq b$ and $u \leq v$.
In this case $F(a, b) = 0$, $F(u, v) = 0$. Then obvious.

Case III. When $t > 2$. We have

$$\frac{t+\frac{4}{3}}{t+\frac{4}{3}+1} = \frac{3t+4}{3t+7} \geq (\frac{t}{t+1})^4. \tag{4.5}$$

Subcase I.$a \geq b$ and $u \geq v$.
we have $F(a, b) = \frac{a^2-b^2}{24}$, $F(u, v) = \frac{u^2-v^2}{24}$.
Now

$$
\begin{aligned}
M(F(a, b), F(u, v), \varphi(t)) &= (\frac{\varphi(t)}{\varphi(t)+1})^{|F(a,b)-F(u,v)|} \\
&\geq (\frac{t+\frac{4}{3}}{t+\frac{4}{3}+1})^{|F(a,b)-F(u,v)|} \\
&= (\frac{3t+4}{3t+7})^{|F(a,b)-F(u,v)|} \\
&= (\frac{3t+4}{3t+7})^{\frac{|(a^2-u^2)-(b^2-v^2)|}{24}} \\
&\geq (\frac{3t+4}{3t+7})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{8}} \\
&\geq (\frac{t}{t+1})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{2}} \qquad \text{(by using (4.5))} \\
&\geq \sqrt{(\frac{t}{t+1})^{|a^2-u^2|}.(\frac{t}{t+1})^{|b^2-v^2|}} \\
&= \min\{(\frac{t}{t+1})^{|a^2-u^2|}, (\frac{t}{t+1})^{|b^2-v^2|}\} \\
&= M(a^2, u^2, t) * M(b^2, v^2, t) \\
&= M(g(a), g(u), t) * M(g(b), g(v), t).
\end{aligned}
$$

Subcase II. $a \leq b$ and $u \geq v$.
we have $F(a, b) = 0$, $F(u, v) = \frac{u^2-v^2}{24}$.
We have $a \leq u$, it then follows that $|u^2 - v^2| \leq |a^2 - u^2|$,
that is, $|u^2 - v^2| \leq |a^2 - u^2| + |b^2 - v^2|$,

$$\text{that is, } \frac{|u^2-v^2|}{24} \leq \frac{1}{24}[|a^2 - u^2| + |b^2 - v^2|]. \tag{4.6}$$

$$
\begin{aligned}
M(F(a, b), F(u, v), \varphi(t)) &= (\frac{\varphi(t)}{\varphi(t)+1})^{|F(a,b)-F(u,v)|} \\
&\geq (\frac{t+\frac{4}{3}}{t+\frac{4}{3}+1})^{|F(a,b)-F(u,v)|} \\
&= (\frac{3t+4}{3t+7})^{|F(u,v)|} \qquad \text{(since F(x,y) = 0)} \\
&= (\frac{3t+4}{3t+7})^{\frac{|u^2)-v^2)|}{24}} \\
&\geq (\frac{3t+4}{3t+7})^{\frac{|(a^2-u^2)-(b^2-v^2)|}{24}} \qquad \text{(by using (4.6))} \\
&\geq (\frac{3t+4}{3t+7})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{8}} \\
&\geq (\frac{t}{t+1})^{\frac{|(a^2-u^2)|+|(b^2-v^2)|}{2}} \qquad \text{(by using (4.5))} \\
&\geq \sqrt{(\frac{t}{t+1})^{|a^2-u^2|}.(\frac{t}{t+1})^{|b^2-v^2|}}
\end{aligned}
$$

$$\begin{aligned}&= \min\{(\frac{t}{t+1})^{|a^2-u^2|}, (\frac{t}{t+1})^{|b^2-v^2|}\}\\ &= M(a^2, u^2, t) * M(b^2, v^2, t)\\ &= M(g(a), g(u), t) * M(g(b), g(v), t).\end{aligned}$$

Subcase III. $a \leq b$ and $u \leq v$.
Here we have $F(a, b) = 0$, $F(u, v) = 0$. Then obvious.
Hence (3.10) holds.
Then, by applying of the Theorem 3.2, we conclude that the pair (g, F) has a coupled coincidence point which is $(0, 0)$ here.

Remark 4.2 Since the function φ in the example is a member Φ and not a member of Φ_w, it follows that Corollary 3.3 is properly included in Theorem 3.2. Again (g, F) is a compatible pair which is not commuting, Theorem 3.4 is properly included in Theorem 3.2. Corollary 3.5 is trivially included in Theorem 3.2, g is different the identity function.

Conclusion. We have used *Hadžić* type t-norm in this paper. Its properties have been used in the proof of Lemma 3.1 which is instrumental to the proof of the main result. It can be further investigated whether these results are valid with some other types of t-norms.

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